SURFACE AND VOLUME CURRENT IN A WIRE

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.5.

A couple of examples of surface and volume current.

If we have a wire of radius a which carries a current I, what is the surface current K if the current in the wire is spread uniformly over the surface of the wire?

Since K is defined as the charge per unit width per unit time, we need the width over which the surface current is spread. This is just the circumference of the wire, so

$$K = \frac{I}{2\pi a} \tag{1}$$

Now suppose the current is distributed in the volume of the wire such that volume current density is inversely proportional to the distance from the axis. That is,

$$J = \frac{A}{r} \tag{2}$$

for some constant A. We need the total flow across the cross-sectional area of the wire to be the total current I, so we must have, using cylindrical coordinates:

$$\begin{aligned} I &= \int_0^a \int_0^{2\pi} J d\phi r dr && (3)\\ &= 2\pi A a && (4)\\ A &= \frac{I}{2\pi a} && (5) \end{aligned}$$

Thus the volume current density is

$$J = \frac{I}{2\pi a r} \tag{6}$$