

BIOT-SAVART LAW - CURRENT LOOPS

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.8.

As mentioned earlier, besides feeling a force from an external magnetic field, an electric current also produces its own magnetic field. The experimentally determined rule for calculating this generated magnetic field is known as the Biot-Savart law. For a steady current (one that doesn't vary with time) in a wire, this law can be written as

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{I} \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} dl' \quad (1)$$

where \mathbf{r}' is a location on the wire and \mathbf{r} is the point at which you want to determine the magnetic field. The constant μ_0 is known as the permeability of free space, and is the magnetic analogue to ϵ_0 in electrostatics. Its value is

$$\mu_0 = 1.25663706 \times 10^{-6} \text{m kg s}^{-2} \text{Amp}^{-2} \quad (2)$$

Again, this law isn't derived from anything more fundamental; it's a generalization of experiment, although the value of μ_0 is fixed at exactly $4\pi \times 10^{-7} \text{m kg s}^{-2} \text{Amp}^{-2}$.

As an example, suppose we want to find the field generated by a steady current travelling round a square loop of side length $2R$, at the centre of the square. We can do this by finding the field generated by a single wire segment first.

Because of the cross product in the integrand, the field will be perpendicular to the plane of the square, so if we call this direction the z axis, and set the edge of the square at $x = R$, we can write the integral as (setting $\mathbf{r} = 0$ since we're interested in the field at the origin):

$$\mathbf{B}_1(\mathbf{r}) = \hat{\mathbf{z}} \frac{I\mu_0}{4\pi} \int \frac{|\mathbf{r} - \mathbf{r}'| \sin \theta}{|\mathbf{r} - \mathbf{r}'|^3} dl' \quad (3)$$

$$= \hat{\mathbf{z}} \frac{I\mu_0}{4\pi} \int_{-R}^R \frac{R}{(R^2 + y^2)^{3/2}} dy \quad (4)$$

$$= \hat{\mathbf{z}} \frac{\sqrt{2}I\mu_0}{4\pi R} \quad (5)$$

where θ is the angle between \mathbf{I} and $\mathbf{r} - \mathbf{r}'$, so that $\sin \theta = R/|\mathbf{r} - \mathbf{r}'|$.

By symmetry, the contribution from all 4 sides is equal, so we get for the total field

$$\mathbf{B} = 4\mathbf{B}_1 = \hat{\mathbf{z}} \frac{\sqrt{2}I\mu_0}{\pi R} \quad (6)$$

Now suppose we have a current loop consisting of a regular polygon with n sides. In this case, each side subtends an angle of $2\pi/n$, so if we align one side parallel to the y axis at $x = R$, this side will extend from an angle of $-\pi/n$ to $+\pi/n$, and will have a length of $2R \tan \frac{\pi}{n}$. Now the integral for a single side is

$$\mathbf{B}_1(0) = \hat{\mathbf{z}} \frac{I\mu_0}{4\pi} \int_{-R \tan \frac{\pi}{n}}^{R \tan \frac{\pi}{n}} \frac{R}{(R^2 + y^2)^{3/2}} dy \quad (7)$$

$$= \hat{\mathbf{z}} \frac{I\mu_0}{4\pi} \frac{2R \tan \frac{\pi}{n}}{R \sqrt{R^2 + (R \tan \frac{\pi}{n})^2}} \quad (8)$$

$$= \hat{\mathbf{z}} \frac{I\mu_0}{2\pi R} \sin \frac{\pi}{n} \quad (9)$$

Each of the n sides will still contribute an equal amount, so the total field is

$$\mathbf{B} = n\mathbf{B}_1 = \hat{\mathbf{z}} \frac{nI\mu_0}{2\pi R} \sin \frac{\pi}{n} \quad (10)$$

As $n \rightarrow \infty$, this formula should give us the field due to a circular loop. In this limit, we can approximate the sine by the first term in its Taylor expansion $\sin \frac{\pi}{n} \approx \frac{\pi}{n}$ so we get

$$\lim_{n \rightarrow \infty} \mathbf{B} = \hat{\mathbf{z}} \frac{I\mu_0}{2R} \quad (11)$$

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