

BIOT-SAVART LAW: A COUPLE MORE EXAMPLES

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.9.

Here are a couple more examples of calculating the magnetic field due to a steady current in a wire.

For the first configuration (bear with me as my picture drawing skills aren't up to much so I'll describe this in words), we have a loop defined as follows.

Starting at $x = a, y = 0$, draw a circular arc of radius a through an angle of $\pi/2$ up to $x = 0, y = a$. Then draw a line out to $x = 0, y = b$, then another circular arc of radius b back down to $x = b, y = 0$, then finally join this up with a line to $x = a, y = 0$ to complete the circuit. Find the field at the centre of the circles defining the arcs.

We apply the Biot-Savart law

$$(1) \quad \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{I} \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} dl'$$

where \mathbf{r}' is a location on the wire and \mathbf{r} is the point at which you want to determine the magnetic field. Along the two radial segments \mathbf{I} is parallel to $\mathbf{r} - \mathbf{r}'$ so the cross product is zero, thus the only contributions come from the two arcs.

From the earlier post, we saw that the field due to a complete circular loop of radius R is, where the current flows in a counter-clockwise direction as seen from above

$$(2) \quad \mathbf{B} = \hat{\mathbf{z}} \frac{I\mu_0}{2R}$$

Each arc is $\frac{1}{4}$ of a circle, and the current flows in the opposite direction in the outer arc, so the net field is

$$(3) \quad \mathbf{B} = \hat{\mathbf{z}} \frac{I\mu_0}{8a} - \hat{\mathbf{z}} \frac{I\mu_0}{8b}$$
$$(4) \quad = \hat{\mathbf{z}} \frac{I\mu_0}{8} \left(\frac{1}{a} - \frac{1}{b} \right)$$

Now consider a semi-circular arc of radius R extending from $\theta = +\frac{\pi}{2}$ to $\theta = \frac{3\pi}{2}$. Join each end of this arc to a horizontal wire extending to infinity in the x direction (so we have infinite wires along the lines $y = \pm R$, starting at the y axis and extending out to positive infinity in the x direction). The current flows in along the bottom wire and out along the top wire. We want the field at the centre of circle defining the semi-circular arc.

The field due to the semi-circle is (negative since the current is flowing clockwise)

$$(5) \quad \mathbf{B}_s = -\hat{\mathbf{z}} \frac{I\mu_0}{4R}$$

For the two straight segments, we can use the same approach as in the earlier post. For the bottom wire, we get

$$(6) \quad \mathbf{B}_b = -\hat{\mathbf{z}} \frac{I\mu_0}{4\pi} \int_0^\infty \frac{R}{(R^2 + x^2)^{3/2}} dx$$

$$(7) \quad = -\hat{\mathbf{z}} \frac{I\mu_0}{4\pi R}$$

By symmetry, the top wire contributes the same amount, so the total field is

$$(8) \quad \mathbf{B} = \mathbf{B}_s + 2\mathbf{B}_b$$

$$(9) \quad = -\hat{\mathbf{z}} \frac{I\mu_0}{4R} \left(1 + \frac{2}{\pi} \right)$$