

## BIOT-SAVART LAW: A COUPLE MORE EXAMPLES

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.9.

Here are a couple more examples of calculating the magnetic field due to a steady current in a wire.

For the first configuration (bear with me as my picture drawing skills aren't up to much so I'll describe this in words), we have a loop defined as follows.

Starting at  $x = a, y = 0$ , draw a circular arc of radius  $a$  through an angle of  $\pi/2$  up to  $x = 0, y = a$ . Then draw a line out to  $x = 0, y = b$ , then another circular arc of radius  $b$  back down to  $x = b, y = 0$ , then finally join this up with a line to  $x = a, y = 0$  to complete the circuit. Find the field at the centre of the circles defining the arcs.

We apply the Biot-Savart law

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{I} \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} dl' \quad (1)$$

where  $\mathbf{r}'$  is a location on the wire and  $\mathbf{r}$  is the point at which you want to determine the magnetic field. Along the two radial segments  $\mathbf{I}$  is parallel to  $\mathbf{r} - \mathbf{r}'$  so the cross product is zero, thus the only contributions come from the two arcs.

From the earlier post, we saw that the field due to a complete circular loop of radius  $R$  is, where the current flows in a counter-clockwise direction as seen from above

$$\mathbf{B} = \hat{\mathbf{z}} \frac{I\mu_0}{2R} \quad (2)$$

Each arc is  $\frac{1}{4}$  of a circle, and the current flows in the opposite direction in the outer arc, so the net field is

$$\mathbf{B} = \hat{\mathbf{z}} \frac{I\mu_0}{8a} - \hat{\mathbf{z}} \frac{I\mu_0}{8b} \quad (3)$$

$$= \hat{\mathbf{z}} \frac{I\mu_0}{8} \left( \frac{1}{a} - \frac{1}{b} \right) \quad (4)$$

Now consider a semi-circular arc of radius  $R$  extending from  $\theta = +\frac{\pi}{2}$  to  $\theta = \frac{3\pi}{2}$ . Join each end of this arc to a horizontal wire extending to infinity in the  $x$  direction (so we have infinite wires along the lines  $y = \pm R$ , starting at the  $y$  axis and extending out to positive infinity in the  $x$  direction). The current flows in along the bottom wire and out along the top wire. We want the field at the centre of circle defining the semi-circular arc.

The field due to the semi-circle is (negative since the current is flowing clockwise)

$$\mathbf{B}_s = -\hat{\mathbf{z}} \frac{I\mu_0}{4R} \quad (5)$$

For the two straight segments, we can use the same approach as in the earlier post. For the bottom wire, we get

$$\mathbf{B}_b = -\hat{\mathbf{z}} \frac{I\mu_0}{4\pi} \int_0^\infty \frac{R}{(R^2 + x^2)^{3/2}} dx \quad (6)$$

$$= -\hat{\mathbf{z}} \frac{I\mu_0}{4\pi R} \quad (7)$$

By symmetry, the top wire contributes the same amount, so the total field is

$$\mathbf{B} = \mathbf{B}_s + 2\mathbf{B}_b \quad (8)$$

$$= -\hat{\mathbf{z}} \frac{I\mu_0}{4R} \left( 1 + \frac{2}{\pi} \right) \quad (9)$$