BIOT-SAVART LAW: A COUPLE MORE EXAMPLES

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.9.

Here are a couple more examples of calculating the magnetic field due to a steady current in a wire.

For the first configuration (bear with me as my picture drawing skills aren't up to much so I'll describe this in words), we have a loop defined as follows.

Starting at $x=a$, $y=0$, draw a circular arc of radius a through an angle of $\pi/2$ up to $x=0$, $y=a$. Then draw a line out to $x=0$, $y=b$, then another circular arc of radius b back down to $x=b$, $y=0$, then finally join this up with a line to $x=a$, $y=0$ to complete the circuit. Find the field at the centre of the circles defining the arcs.

We apply the Biot-Savart law

$$\mathbf{B}(\mathbf{r})=\frac{\mu_0}{4\pi}\int\frac{\mathbf{I}\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}dl' \tag{1}$$

where $\mathbf{r}'$ is a location on the wire and $\mathbf{r}$ is the point at which you want to determine the magnetic field. Along the two radial segments $\mathbf{I}$ is parallel to $\mathbf{r}-\mathbf{r}'$ so the cross product is zero, thus the only contributions come from the two arcs.

From the earlier post, we saw that the field due to a complete circular loop of radius R is, where the current flows in a counter-clockwise direction as seen from above

$$\mathbf{B}=\hat{\mathbf{z}}\frac{I\mu_0}{2R} \tag{2}$$

Each arc is $\frac{1}{4}$ of a circle, and the current flows in the opposite direction in the outer arc, so the net field is

$$\begin{aligned}\mathbf{B} &= \hat{\mathbf{z}}\frac{I\mu_0}{8a}-\hat{\mathbf{z}}\frac{I\mu_0}{8b} &(3)\\ &= \hat{\mathbf{z}}\frac{I\mu_0}{8}\left(\frac{1}{a}-\frac{1}{b}\right) &(4)\end{aligned}$$

Now consider a semi-circular arc of radius R extending from $\theta=+\frac{\pi}{2}$ to $\theta=\frac{3\pi}{2}$. Join each end of this arc to a horizontal wire extending to infinity in the x direction (so we have infinite wires along the lines $y=\pm R$, starting at the y axis and extending out to positive infinity in the x direction). The current flows in along the bottom wire and out along the top wire. We want the field at the centre of circle defining the semi-circular arc.

The field due to the semi-circle is (negative since the current is flowing clockwise)

$$\mathbf{B}_s=-\hat{\mathbf{z}}\frac{I\mu_0}{4R} \tag{5}$$

For the two straight segments, we can use the same approach as in the earlier post. For the bottom wire, we get

$$\begin{aligned}\mathbf{B}_b &= -\hat{\mathbf{z}}\frac{I\mu_0}{4\pi}\int_0^{\infty}\frac{R}{\left(R^2+x^2\right)^{3/2}}dx && (6)\\ &= -\hat{\mathbf{z}}\frac{I\mu_0}{4\pi R} && (7)\end{aligned}$$

By symmetry, the top wire contributes the same amount, so the total field is

$$\begin{aligned}\mathbf{B} &= \mathbf{B}_s+2\mathbf{B}_b && (8)\\ &= -\hat{\mathbf{z}}\frac{I\mu_0}{4R}\left(1+\frac{2}{\pi}\right) && (9)\end{aligned}$$