ELECTRIC AND MAGNETIC FORCES IN TWO CHARGED WIRES

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.12.

The electric field a distance d from an infinite line of charge (linear charge density λ) is

$$E=\frac{1}{4\pi\epsilon_0}\frac{2\lambda}{d} \tag{1}$$

For positive charge, this field points radially outwards from the wire.

Now if we move this line of charge at speed v, thus creating a current $I=\lambda v$, it will produce a magnetic field

$$\begin{aligned} B &= \frac{I\mu_0}{2\pi d} && (2)\\ &= \frac{\lambda v\mu_0}{2\pi d} && (3) \end{aligned}$$

The field will circle the line in a direction determined by the right-hand rule.

Now suppose we place an identical line of charge, moving at the same speed, a distance d from the first one. Since both lines carry the same sign of charge, the electric force between them is repulsive. From the Biot-Savart law and the right hand rule, we can see that the magnetic force is attractive. Can we adjust v so that the electric and magnetic forces balance each other?

For a unit length of wire, the electric force is

$$F_E=\frac{1}{4\pi\epsilon_0}\frac{2\lambda^2}{d} \tag{4}$$

and the magnetic force is

$$F_B=\frac{(\lambda v)^2\mu_0}{2\pi d} \tag{5}$$

Setting these two equal to each other, we get

$$v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \tag{6}$$

$$= \frac{1}{\sqrt{(8.85418782 \times 10^{-12})(1.25663706 \times 10^{-6})}} \tag{7}$$

$$= 2.997924 \times 10^{8} \mathrm{m\ s}^{-1} \tag{8}$$

This happens to be the speed of light, so it's not possible to get the charge moving fast enough for the magnetic force to balance the electric force.

It's not a coincidence that $\frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c$; in fact this quantity comes out of Maxwell's equations as the speed of electromagnetic waves, and is what led Einstein to postulate that c is a universal constant and thus create his theory of special relativity. We'll get to this eventually.

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