AMPÈRE'S LAW: SLAB OF CURRENT

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.14.

Here's another simple example of Ampère's law. We have an infinite slab parallel to the xy plane, extending from $z=-a$ to $z=+a$, and carrying a volume current density J in the $+x$ direction. What is the magnetic field both inside and outside the slab?

First, we need to find the direction of the field. From the Biot-Savart law, we know the field must be perpendicular to the current, so there can be no field in the x direction. Now if we consider a thin line of current parallel to the x axis, we know that the field loops around the line so that it forms a counterclockwise cylinder when viewed down the x axis. By the symmetry of the setup, the z component of the field in the xz plane due to a line of current extending along the line $z=b,\ y=c$ (for some constants b and c) is cancelled by the z component from the line of current along $z=b,\ y=-c$, so the net z component of the field in the xz plane must be zero. Since the slab is infinite in extent in the xy plane, we can apply this symmetry argument to all planes parallel to the xz plane, so there is no z component of the field anywhere. Thus the field is entirely in the y direction.

To find its magnitude, we'll consider first the region outside the slab. The field is in the $-y$ direction above the slab and in the $+y$ direction below. Take a rectangular loop cutting through the slab, with one pair of sides having length 1, parallel to the yz plane, centred on the xy plane, and extending to $z=\pm d$ where $d>a$. Using Ampère's law we get

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0\int \mathbf{J}\cdot d\mathbf{a} \tag{0.1}$$

$$2B = \mu_0 2aJ \tag{0.2}$$

$$B = \mu_0 aJ \tag{0.3}$$

Inside the slab, we can observe that in the xy plane, $B=0$ since any contribution to the y component of B from an element at $z=b$ is cancelled by the contribution from $z=-b$. Also from symmetry arguments, the field at $z=\pm d$ is $\pm B_y$ where B_y is the y component to be determined. Using another loop like the previous one except this time with $d<a$, we get

$$2\left|B_y\right| = \mu_0 2dJ \tag{0.4}$$

$$\left|B_y\right| = \mu_0 dJ \tag{0.5}$$

In terms of z, this gives

$$\mathbf{B} = -\mu_0 Jz\hat{\mathbf{y}} \tag{0.6}$$

for $-a \le z \le a$.

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