FORCE BETWEEN TWO SHEETS OF CURRENT

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.16.

As an example of Ampère's law applied to surface currents, suppose we have two parallel, infinite sheets of charge parallel to the xy plane, with the upper sheet having a charge density of $+\sigma$ and the lower sheet having $-\sigma$. If the sheets are both moving with speed v in the $+x$ direction, what is the magnetic field everywhere?

This problem is similar to that of the slab of current that we treated earlier. Consider first the upper sheet. By the same arguments as in the earlier case, the field above the sheet is in the $-y$ direction, and below the sheet is in the $+y$ direction. The magnitude of the field is obtained from Ampère's law, using a surface current of $\mathbf{K}=\sigma v\hat{\mathbf{x}}$. Using an integration loop with a side-length of 1 unit, we get

$$\oint\mathbf{B}_u\cdot d\mathbf{l} = \mu_0\int\mathbf{K}\cdot d\mathbf{n} \tag{1}$$

$$2B_u = \mu_0\sigma v \tag{2}$$

$$\mathbf{B}_u = \mp\frac{\mu_0\sigma v}{2}\hat{\mathbf{y}} \tag{3}$$

with the plus sign for points below the sheet, and the minus sign for points above. In the first line, $d\mathbf{n}$ represents a normal to a line segment in the cross section of one of the sheets of charge, and we integrate over this cross section of length 1 unit. $d\mathbf{n}$ is essentially $\hat{\mathbf{x}}d\ell$ where $d\ell$ is the length of the infinitesimal line segment. This line segment is not the same as $d\mathbf{l}$ in the integral of magnetic field, since that integral is a line integral around a loop.

For the lower sheet, since the charge density is negative, the field is reversed, so we get

$$\mathbf{B}_l = \pm\frac{\mu_0\sigma v}{2}\hat{\mathbf{y}} \tag{4}$$

Thus above the top sheet and below the bottom sheet, the net field is zero. Between the sheets (if you'll pardon the phrase), the two fields add, and we get

$$\mathbf{B} = \mu_0 \sigma v \hat{\mathbf{y}} \tag{5}$$

The force per unit area on the top sheet can be found from the Lorentz force law. For a unit area, we have

$$\begin{aligned} \mathbf{F}_{mag} &= \sigma v \hat{\mathbf{x}} \times \mathbf{B}_l && (6) \\ &= \frac{\mu_0 (\sigma v)^2}{2} \hat{\mathbf{z}} && (7) \end{aligned}$$

That is, the force pushes the sheets apart.

The electrical force is attractive, since the two sheets carry opposite charges, and the electric field due to an infinite plane of charge is

$$\mathbf{E} = -\frac{\sigma}{2\varepsilon_0} \hat{\mathbf{z}} \tag{8}$$

The force per unit area is therefore

$$\begin{aligned} \mathbf{F}_{elec} &= \sigma \mathbf{E} && (9) \\ &= -\frac{\sigma^2}{2\varepsilon_0} \hat{\mathbf{z}} && (10) \end{aligned}$$

If we try to balance the electric and magnetic forces, we must have

$$\begin{aligned} \frac{\mu_0 (\sigma v)^2}{2} &= \frac{\sigma^2}{2\varepsilon_0} && (11) \\ v &= \frac{1}{\sqrt{\mu_0 \varepsilon_0}} && (12) \\ &= c && (13) \end{aligned}$$

(that is, the speed of light). Not surprisingly, this is the same result that we got for two line charges.

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