ELECTRON SPEED IN A COPPER WIRE

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.19.

It's interesting to work out some actual numbers for electric and magnetic properties of a real material, such as copper. The density of copper is 8.96 g cm^{-3} and its atomic weight is 63.546 so the number of atoms per cubic cm is

$$n_{Cu} = \frac{8.96}{63.546} \times 6.022142 \times 10^{23} \tag{1}$$

$$= 8.49123 \times 10^{22} \tag{2}$$

Assuming 2 free electrons per atom the density of free charges is therefore $1.69825 \times 10^{23}\ \text{cm}^{-3}$.

Suppose we have a copper wire 1 mm in diameter carrying a current of 1 Amp. One amp is one coulomb per second, so that means we have 6.2415×10^{18} electrons per second passing a given point in the wire. Given the charge density, we can work out the linear charge density in the wire. A centimetre length of the wire has a volume of $\pi (0.05)^2 = 7.854 \times 10^{-3}\ \text{cm}^3$, so the linear charge density is

$$\lambda = 1.69825 \times 10^{23} \times 7.854 \times 10^{-3} \tag{3}$$

$$= 1.3338 \times 10^{21}\ \text{cm}^{-1} \tag{4}$$

Since we require 6.2415×10^{18} electrons per second, the speed at which charge moves down the wire is

$$v = \frac{6.2415 \times 10^{18}}{1.3338 \times 10^{21}} \tag{5}$$

$$= 4.6795 \times 10^{-3}\ \text{cm} \cdot \text{s}^{-1} \tag{6}$$

In other words, it takes about 3.5 minutes for an electron to move 1 cm, which is extremely slow. This seems paradoxical; after all, when you flip a light switch it doesn't take several hours for the light to come on, as it would if we had to wait for electrons to trickle all the way from the switch to the light bulb.

The answer to this paradox is that it's not the electrons themselves that carry the energy (or the signal, in the case of, say, a computer's network connection or a telephone line), it's the electromagnetic wave that travels down the wire. In a vacuum, these waves travel at the speed of light, while in unshielded copper, the wave travels at up to 97% of the speed of light. For more details, see this Wikipedia article.

If we place 2 such wires parallel to each other at a distance of 1 cm, we can work out the force between them. Since they are electrically neutral, there is no electric force, but there is a magnetic force. The magnetic field due to one wire is

$$B = \frac{I\mu_0}{2\pi d} \tag{7}$$

For a current of 1 amp and distance of 0.01 m, the field is (using MKS units)

$$B = \frac{(1)\,4\pi \times 10^{-7}}{2\pi\,(0.01)} \tag{8}$$

$$= 2 \times 10^{-5}\text{T} \tag{9}$$

For a unit length of the other wire (1 metre), the force is then

$$F = IB \tag{10}$$

$$= 2 \times 10^{-5}\text{N} \cdot \text{m}^{-1} \tag{11}$$

If we had a linear charge density as above consisting of electrons only (no positive charges), then the electric field from one wire is (using MKS units)

$$E = \frac{2\lambda}{4\pi\epsilon_0 d} \tag{12}$$

$$= \frac{1.3338 \times 10^{23} \times 1.6022 \times 10^{-19}}{2\pi\,(8.854 \times 10^{-12})\,(0.01)} \tag{13}$$

$$= 3.8414 \times 10^{16}\text{V} \cdot \text{m}^{-1} \tag{14}$$

The force on a metre of the other line charge is then

$$F = \lambda E \tag{15}$$

$$= \left(1.3338 \times 10^{23} \times 1.6022 \times 10^{-19}\right)\left(3.8414 \times 10^{16}\right) \tag{16}$$

$$= 8.21 \times 10^{20}\text{N} \cdot \text{m}^{-1} \tag{17}$$

This is around 4×10^{25} times greater than the magnetic force, so clearly the electric force is a lot stronger.