AMPÈRE'S LAW AND THE STEADY CURRENT ASSUMPTION

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.20.

This is a bit of a footnote to the post on the surface used to evaluate the integral in Ampère's law. It was pointed out there that one of the assumptions made in deriving Ampère's law was that currents were steady, in particular that $\nabla\cdot\mathbf{J}=0$. This assumption makes the differential form of the law consistent. This form states that

$$\nabla\times\mathbf{B}=\mu_0\mathbf{J} \tag{1}$$

An identity from vector calculus states that, for any vector field, $\nabla\cdot(\nabla\times\mathbf{A})=0$. This is true in the steady-current case, but breaks down in the more general case where $\nabla\cdot\mathbf{J}=\partial\rho/\partial t$, that is, where the current is responsible for redistributing the charge distribution.

There is a related problem with the electric field equation $\nabla\times\mathbf{E}=0$. Although this doesn't pose any problems mathematically, since the identity $\nabla\cdot(\nabla\times\mathbf{E})=0$ is still true. However, the curl-free field equation was derived from Coulomb's law, which in turn gave rise to the fact that in electrostatics, the field can be written as the gradient of a potential, and the vector identity $\nabla\times\nabla\Phi=0$ then gives us the zero curl. Once charges start to move, they generate magnetic fields, and as we'll see (eventually), varying magnetic fields can affect the electric field and give rise to a non-zero curl.