

MAGNETIC VECTOR POTENTIAL OF CONSTANT FIELD

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.24.

The magnetic vector potential \mathbf{A} for a uniform (that is, constant) magnetic field \mathbf{B} is

$$(1) \quad \mathbf{A} = -\frac{1}{2}\mathbf{r} \times \mathbf{B}$$

To check this, we calculate its divergence and curl. For the divergence, using a vector calculus identity

$$(2) \quad \nabla \cdot \mathbf{A} = -\frac{1}{2}[\mathbf{B} \cdot (\nabla \times \mathbf{r}) - \mathbf{r} \cdot (\nabla \times \mathbf{B})]$$

The last term is zero since \mathbf{B} is a constant, and $\nabla \times \mathbf{r} = 0$ as can be checked by direct calculation, so $\nabla \cdot \mathbf{A} = 0$ which is one condition required of \mathbf{A} .

For the curl, we get (omitting terms involving a derivative of \mathbf{B} , which are all zero):

$$(3) \quad \nabla \times \mathbf{A} = -\frac{1}{2}\nabla \times (\mathbf{r} \times \mathbf{B})$$

$$(4) \quad -\frac{1}{2}[(\mathbf{B} \cdot \nabla)\mathbf{r} - \mathbf{B}(\nabla \cdot \mathbf{r})]$$

$$(5) \quad = -\frac{1}{2}[\mathbf{B} - 3\mathbf{B}]$$

$$(6) \quad = \mathbf{B}$$

To get the third line from the second, here's a sample calculation of the x component of the first term:

$$(7) \quad (\mathbf{B} \cdot \nabla)x\hat{\mathbf{x}} = (B_x\partial_x + B_y\partial_y + B_z\partial_z)x\hat{\mathbf{x}}$$

$$(8) \quad = B_x\hat{\mathbf{x}}$$

The y and z components work the same way. We've also used $\nabla \cdot \mathbf{r} = 3$ as can be checked by direct calculation.

Thus the div and curl of \mathbf{A} give the correct values. It's not unique, since we can add any vector field to \mathbf{A} so long as its div and curl are both zero. This is true of any constant field, so in general

$$(9) \quad \mathbf{A} = -\frac{1}{2}\mathbf{r} \times \mathbf{B} + \mathbf{C}$$

where \mathbf{C} is constant over all space.

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