MAGNETIC VECTOR POTENTIAL OF CONSTANT FIELD

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.24.

The magnetic vector potential $\mathbf{A}$ for a uniform (that is, constant) magnetic field $\mathbf{B}$ is

$$\mathbf{A}=-\frac{1}{2}\mathbf{r}\times\mathbf{B} \tag{1}$$

To check this, we calculate its divergence and curl. For the divergence, using a vector calculus identity

$$\nabla\cdot\mathbf{A}=-\frac{1}{2}\left[\mathbf{B}\cdot(\nabla\times\mathbf{r})-\mathbf{r}\cdot(\nabla\times\mathbf{B})\right] \tag{2}$$

The last term is zero since $\mathbf{B}$ is a constant, and $\nabla\times\mathbf{r}=0$ as can be checked by direct calculation, so $\nabla\cdot\mathbf{A}=0$ which is one condition required of $\mathbf{A}$.

For the curl, we get (omitting terms involving a derivative of $\mathbf{B}$, which are all zero):

$$\begin{aligned}
\nabla\times\mathbf{A} &= -\frac{1}{2}\nabla\times(\mathbf{r}\times\mathbf{B}) && (3)\\
& \quad -\frac{1}{2}\left[(\mathbf{B}\cdot\nabla)\mathbf{r}-\mathbf{B}(\nabla\cdot\mathbf{r})\right] && (4)\\
&= -\frac{1}{2}\left[\mathbf{B}-3\mathbf{B}\right] && (5)\\
&= \mathbf{B} && (6)
\end{aligned}$$

To get the third line from the second, here's a sample calculation of the x component of the first term:

$$\begin{aligned}
(\mathbf{B}\cdot\nabla)x\hat{\mathbf{x}} &= (B_x\partial_x+B_y\partial_y+B_z\partial_z)x\hat{\mathbf{x}} && (7)\\
&= B_x\hat{\mathbf{x}} && (8)
\end{aligned}$$

The y and z components work the same way. We've also used $\nabla\cdot\mathbf{r}=3$ as can be checked by direct calculation.

Thus the div and curl of **A** give the correct values. It's not unique, since we can add any vector field to **A** so long as its div and curl are both zero. This is true of any constant field, so in general

$$\mathbf{A}=-\frac{1}{2}\mathbf{r}\times\mathbf{B}+\mathbf{C} \tag{9}$$

where **C** is constant over all space.

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