MAGNETIC VECTOR POTENTIAL: SHEET OF CURRENT

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.26.

Another example of calculating the magnetic vector potential in a case where the current extends to infinity. We consider a uniform sheet of current in the xy plane, carrying surface current density $K\hat{\mathbf{x}}$. Using the same argument as in the case of a slab of current, the magnetic field due to this current is

$$\mathbf{B}=\begin{cases}-\frac{\mu_0}{2}K\hat{\mathbf{y}} & z>0\\ \frac{\mu_0}{2}K\hat{\mathbf{y}} & z<0\end{cases} \tag{1}$$

Note that the field is independent of the distance from the sheet of current.

We can work out the potential by applying Stokes's theorem. From the definition of $\mathbf{A}$, we know that $\nabla\times\mathbf{A}=\mathbf{B}$, so if we define a closed loop then we have

$$\oint\mathbf{A}\cdot d\mathbf{l}=\int_A\mathbf{B}\cdot d\mathbf{a} \tag{2}$$

where the integral on the LHS is a line integral around the loop and the integral on the right is over the area A enclosed by the loop. For a flat loop, the direction of integration on the LHS is such that the right hand rule gives a vector pointing in the same direction as $d\mathbf{a}$ on the RHS.

We need to be careful in defining directions for the area and line integrals to get the signs right. We'll take as our area A a rectangle above the xy plane and parallel to the xz plane. The sides of the rectangle parallel to the xy plane are of length 1, with the lower side at a distance a and the upper side at a distance b from this plane. The normal to the area points in the $\hat{\mathbf{y}}$ direction Since $\mathbf{B}$ is uniform everywhere above the plane, we get

$$\int_A\mathbf{B}\cdot d\mathbf{a}=-\frac{\mu_0}{2}K(b-a) \tag{3}$$

For the line integral, the path around the rectangle is clockwise when looking in the $+y$ direction, and by symmetry, the integral of $\mathbf{A}\cdot d\mathbf{l}$ along the vertical sides cancels out, so

$$\oint\mathbf{A}\cdot d\mathbf{l}=A_x(b)-A_x(a) \tag{4}$$

Comparing these two, a reasonable candidate is

$$\mathbf{A}=-\frac{\mu_0}{2}Kz\hat{\mathbf{x}} \tag{5}$$

We can check this by finding the div and curl, as usual:

$$\nabla\cdot\mathbf{A} = 0 \tag{6}$$

$$\nabla\times\mathbf{A} = -\frac{\mu_0}{2}K\hat{\mathbf{y}} \tag{7}$$

Below the sheet of current, the sign is reversed so we have

$$\mathbf{A}=\frac{\mu_0}{2}Kz\hat{\mathbf{x}} \tag{8}$$