

MAGNETIC VECTOR POTENTIAL: SHEET OF CURRENT

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.26.

Another example of calculating the magnetic vector potential in a case where the current extends to infinity. We consider a uniform sheet of current in the xy plane, carrying surface current density $K\hat{x}$. Using the same argument as in the case of a slab of current, the magnetic field due to this current is

$$(0.1) \quad \mathbf{B} = \begin{cases} -\frac{\mu_0}{2}K\hat{y} & z > 0 \\ \frac{\mu_0}{2}K\hat{y} & z < 0 \end{cases}$$

Note that the field is independent of the distance from the sheet of current.

We can work out the potential by applying Stokes's theorem. From the definition of \mathbf{A} , we know that $\nabla \times \mathbf{A} = \mathbf{B}$, so if we define a closed loop then we have

$$(0.2) \quad \oint \mathbf{A} \cdot d\mathbf{l} = \int_A \mathbf{B} \cdot d\mathbf{a}$$

where the integral on the LHS is a line integral around the loop and the integral on the right is over the area A enclosed by the loop. For a flat loop, the direction of integration on the LHS is such that the right hand rule gives a vector pointing in the same direction as $d\mathbf{a}$ on the RHS.

We need to be careful in defining directions for the area and line integrals to get the signs right. We'll take as our area A a rectangle above the xy plane and parallel to the xz plane. The sides of the rectangle parallel to the xy plane are of length 1, with the lower side at a distance a and the upper side at a distance b from this plane. The normal to the area points in the \hat{y} direction. Since \mathbf{B} is uniform everywhere above the plane, we get

$$(0.3) \quad \int_A \mathbf{B} \cdot d\mathbf{a} = -\frac{\mu_0}{2}K(b-a)$$

For the line integral, the path around the rectangle is clockwise when looking in the $+y$ direction, and by symmetry, the integral of $\mathbf{A} \cdot d\mathbf{l}$ along the vertical sides cancels out, so

$$(0.4) \quad \oint \mathbf{A} \cdot d\mathbf{l} = A_x(b) - A_x(a)$$

Comparing these two, a reasonable candidate is

$$(0.5) \quad \mathbf{A} = -\frac{\mu_0}{2} Kz \hat{\mathbf{x}}$$

We can check this by finding the div and curl, as usual:

$$(0.6) \quad \nabla \cdot \mathbf{A} = 0$$

$$(0.7) \quad \nabla \times \mathbf{A} = -\frac{\mu_0}{2} K \hat{\mathbf{y}}$$

Below the sheet of current, the sign is reversed so we have

$$(0.8) \quad \mathbf{A} = \frac{\mu_0}{2} Kz \hat{\mathbf{x}}$$