

MAGNETIC SCALAR POTENTIAL

Link to: [physicspages home page](#).

To leave a comment or report an error, please use the [auxiliary blog](#).

Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.28.

Since the electric field can be expressed as the gradient of a scalar potential ($\mathbf{E} = -\nabla V$), it's natural to wonder if the magnetic field could be written as the gradient of a *scalar* potential, as well as the curl of a vector potential. That is, we would like to try

$$\mathbf{B} = -\nabla U \quad (1)$$

for some function U . This has one obvious flaw, in that Ampère's law states that the curl of \mathbf{B} is non-zero in the presence of currents. That is

$$\nabla \times \mathbf{B}(\mathbf{r}) = \mu_0 \mathbf{J}(\mathbf{r}) \quad (2)$$

This is a problem, because the curl of a gradient is always zero (this is an identity from vector calculus). Still, we might try to define a scalar potential in current-free areas. If we try this, we can see one consequence by considering the magnetic field due to the current I in an infinite wire. In cylindrical coordinates, this is

$$\mathbf{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi} \quad (3)$$

If we write this as ∇U , then using the form for the gradient in cylindrical coordinates there will be only a ϕ term:

$$\nabla U = \frac{1}{r} \frac{\partial U}{\partial \phi} \hat{\phi} \quad (4)$$

Now if we apply Ampère's law to a circular path surrounding the wire

$$\int \mathbf{B} \cdot d\mathbf{l} = - \int \nabla U \cdot d\mathbf{l} \quad (5)$$

$$= - \int_0^{2\pi} \left(\frac{1}{r} \frac{\partial U}{\partial \phi} \hat{\phi} \right) \cdot (r \hat{\phi} d\phi) \quad (6)$$

$$= - \int_0^{2\pi} \frac{\partial U}{\partial \phi} d\phi \quad (7)$$

$$= U(0) - U(2\pi) \quad (8)$$

$$= \mu_0 I \quad (9)$$

That is, the difference $U(2\pi) - U(0) \neq 0$ even though $\phi = 0$ and $\phi = 2\pi$ represent the same point. The potential must therefore be a multiple-valued quantity, so it's not of much use in physical situations.

PINGBACKS

Pingback: Magnetic scalar potential: dipole and rotating sphere