MAGNETIC SCALAR POTENTIAL

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.28.

Since the electric field can be expressed as the gradient of a scalar potential ($\mathbf{E}=-\nabla V$), it's natural to wonder if the magnetic field could be written as the gradient of a *scalar* potential, as well as the curl of a vector potential. That is, we would like to try

$$\mathbf{B}=-\nabla U \tag{0.1}$$

for some function U. This has one obvious flaw, in that Ampère's law states that the curl of $\mathbf{B}$ is non-zero in the presence of currents. That is

$$\nabla\times\mathbf{B}(\mathbf{r})=\mu_0\mathbf{J}(\mathbf{r}) \tag{0.2}$$

This is a problem, because the curl of a gradient is always zero (this is an identity from vector calculus). Still, we might try to define a scalar potential in current-free areas. If we try this, we can see one consequence by considering the magnetic field due to the current I in an infinite wire. In cylindrical coordinates, this is

$$\mathbf{B}=\frac{\mu_0 I}{2\pi r}\hat{\phi} \tag{0.3}$$

If we write this as ∇U, then using the form for the gradient in cylindrical coordinates there will be only a ϕ term:

$$\nabla U=\frac{1}{r}\frac{\partial U}{\partial\phi}\hat{\phi} \tag{0.4}$$

Now if we apply Ampère's law to a circular path surrounding the wire

$$\int \mathbf{B}\cdot d\mathbf{l} = -\int \nabla U \cdot d\mathbf{l} \tag{0.5}$$

$$= -\int_0^{2\pi}\left(\frac{1}{r}\frac{\partial U}{\partial\phi}\hat{\phi}\right)\cdot\left(r\hat{\phi}d\phi\right) \tag{0.6}$$

$$= -\int_0^{2\pi}\frac{\partial U}{\partial\phi}d\phi \tag{0.7}$$

$$= U(0)-U(2\pi) \tag{0.8}$$

$$= \mu_0 I \tag{0.9}$$

That is, the difference $U(2\pi)-U(0)\neq 0$ even though $\phi=0$ and $\phi=2\pi$ represent the same point. The potential must therefore be a multiple-valued quantity, so it's not of much use in physical situations.

PINGBACKS

Pingback: Magnetic scalar potential: dipole and rotating sphere