MAGNETOSTATIC BOUNDARY CONDITIONS: A COUPLE OF EXAMPLES

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.31.

Here are a few examples of the magnetic boundary conditions. First consider again the infinite solenoid, which we treated as a surface current travelling around a cylinder. We found that the magnetic field outside the solenoid was zero, and inside it was uniform with a value of $B = \mu_0 nI$ where n is the number of turns of wire per unit length and I is the current. Translated into surface current density, we have $nI = K$, so $B = \mu_0 K$. Thus the discontinuity at the surface is indeed $\mu_0 K$ as our previous analysis showed.

Now consider the rotating spherical shell of charge. We found that the vector potential was

$$\mathbf{A}(r,\theta,\phi) = \begin{cases} \frac{1}{3}\mu_0 R\omega\sigma r\sin\theta\hat{\phi} & r \le R \\ \frac{\mu_0 R^4\omega\sigma}{3}\frac{\sin\theta}{r^2}\hat{\phi} & r \ge R \end{cases} \tag{1}$$

At the surface $r = R$ and it's clear that $\mathbf{A}$ is continuous there. For the normal derivative, we have

$$\partial_r \mathbf{A} = \begin{cases} \frac{1}{3}\mu_0 R\omega\sigma\sin\theta\hat{\phi} & r \le R \\ -\frac{2\mu_0 R^4\omega\sigma}{3}\frac{\sin\theta}{r^3}\hat{\phi} & r \ge R \end{cases} \tag{2}$$

The difference as we cross the shell is (taking the outer minus the inner values):

$$\partial_r(\Delta\mathbf{A}) = -\mu_0 R\omega\sigma\sin\theta\hat{\phi} \tag{3}$$

The linear velocity of a patch of area at angle θ is $R\sin\theta\omega$, so the surface current at that point is $\mathbf{K} = R\sin\theta\omega\sigma\hat{\phi}$. Thus $\partial_r(\Delta\mathbf{A}) = -\mu_0\mathbf{K}$ as required.