

MAGNETOSTATIC BOUNDARY CONDITIONS: A COUPLE OF EXAMPLES

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.31.

Here are a few examples of the magnetic boundary conditions. First consider again the infinite solenoid, which we treated as a surface current traveling around a cylinder. We found that the magnetic field outside the solenoid was zero, and inside it was uniform with a value of $B = \mu_0 n I$ where n is the number of turns of wire per unit length and I is the current. Translated into surface current density, we have $nI = K$, so $B = \mu_0 K$. Thus the discontinuity at the surface is indeed $\mu_0 K$ as our previous analysis showed.

Now consider the rotating spherical shell of charge. We found that the vector potential was

$$(0.1) \quad \mathbf{A}(r, \theta, \phi) = \begin{cases} \frac{1}{3} \mu_0 R \omega \sigma r \sin \theta \hat{\phi} & r \leq R \\ \frac{\mu_0 R^4 \omega \sigma \sin \theta}{3 r^2} \hat{\phi} & r \geq R \end{cases}$$

At the surface $r = R$ and it's clear that \mathbf{A} is continuous there. For the normal derivative, we have

$$(0.2) \quad \partial_r \mathbf{A} = \begin{cases} \frac{1}{3} \mu_0 R \omega \sigma \sin \theta \hat{\phi} & r \leq R \\ -\frac{2 \mu_0 R^4 \omega \sigma \sin \theta}{3 r^3} \hat{\phi} & r \geq R \end{cases}$$

The difference as we cross the shell is (taking the outer minus the inner values):

$$(0.3) \quad \partial_r (\Delta \mathbf{A}) = -\mu_0 R \omega \sigma \sin \theta \hat{\phi}$$

The linear velocity of a patch of area at angle θ is $R \sin \theta \omega$, so the surface current at that point is $\mathbf{K} = R \sin \theta \omega \sigma \hat{\phi}$. Thus $\partial_r (\Delta \mathbf{A}) = -\mu_0 \mathbf{K}$ as required.