MAGNETIC DIPOLE MOMENT OF A CURRENT LOOP

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.34.

Here's a simple example of the magnetic dipole. We've got a circular loop of current I with radius R lying in the xy plane and centered at the origin. The dipole moment is

$$\mathbf{m}=I\mathbf{a} \tag{1}$$

where $\mathbf{a}$ is the vector area of the loop, which is just $\mathbf{a}=\pi R^2\hat{\mathbf{z}}$. Thus

$$\mathbf{m}=\pi IR^2\hat{\mathbf{z}} \tag{2}$$

Using the dipole field as an approximation for points far from the loop, we have

$$\begin{aligned}\mathbf{B}_1 &= \frac{\mu_0}{4\pi r^3}\left[3\left(\mathbf{m}\cdot\hat{\mathbf{r}}\right)\hat{\mathbf{r}}-\mathbf{m}\right] && (3)\\ &= \frac{\mu_0\pi IR^2}{4\pi r^3}\left(3\cos\theta\hat{\mathbf{r}}-\hat{\mathbf{z}}\right) && (4)\\ &= \frac{\mu_0 IR^2}{4r^3}\left(2\cos\theta\hat{\mathbf{r}}+\sin\theta\hat{\boldsymbol{\theta}}\right) && (5)\end{aligned}$$

For points on the z axis this becomes

$$\mathbf{B}_1=\frac{\mu_0 IR^2}{2z^3}\hat{\mathbf{z}} \tag{6}$$

The exact formula for points on the z axis (given as example 5.6 in Griffiths) is

$$\mathbf{B}=\frac{\mu_0 IR^2}{2\left(R^2+z^2\right)^{3/2}}\hat{\mathbf{z}} \tag{7}$$

For $z\gg R$, this gives the same result as the dipole field.

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