MAGNETIC DIPOLE MOMENT OF SPINNING SPHERICAL SHELL

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.36.

Now for the magnetic dipole moment of a spinning spherical shell of charge. We can divide the sphere up into a number of circular loops. Each loop corresponds to a particular value of θ and has radius $R\sin\theta$. The current of each loop is $dI = \omega\sigma R\sin\theta R d\theta$ and the magnetic moment of the loop is

$$d\mathbf{m} = \hat{\mathbf{z}}\pi\left(R\sin\theta\right)^2\omega\sigma R^2\sin\theta d\theta \tag{0.1}$$

The total dipole moment is then

$$\mathbf{m} = \hat{\mathbf{z}}\pi\omega\sigma R^4\int_0^\pi \sin^3\theta d\theta \tag{0.2}$$

$$= \frac{4}{3}\pi\omega\sigma R^4\hat{\mathbf{z}} \tag{0.3}$$

For $r > R$, the potential of the sphere is (Griffiths example 5.11):

$$\mathbf{A} = \frac{\mu_0 R^4\omega\sigma}{3}\frac{\sin\theta}{r^2}\hat{\phi} \tag{0.4}$$

The dipole approximation for the potential is

$$\mathbf{A} = \frac{\mu_0}{4\pi r^2}\mathbf{m}\times\hat{\mathbf{r}} \tag{0.5}$$

$$= \frac{\mu_0 R^4\omega\sigma}{3}\frac{\sin\theta}{r^2}\hat{\phi} \tag{0.6}$$

which follows since θ is the angle between $\hat{\mathbf{z}}$ and $\hat{\mathbf{r}}$ so $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \sin\theta\hat{\phi}$. Thus outside the sphere, the dipole potential is the exact potential.

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