MAGNETIC DIPOLE MOMENT OF SPINNING SPHERICAL SHELL

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.36.

Now for the magnetic dipole moment of a spinning spherical shell of charge. We can divide the sphere up into a number of circular loops. Each loop corresponds to a particular value of θ and has radius $R\sin\theta$. The current of each loop is $dI=\omega\sigma R\sin\theta Rd\theta$ and the magnetic moment of the loop is

$$d\mathbf{m}=\hat{\mathbf{z}}\pi\left(R\sin\theta\right)^{2}\omega\sigma R^{2}\sin\theta d\theta \tag{1}$$

The total dipole moment is then

$$\begin{aligned}\mathbf{m} &= \hat{\mathbf{z}}\pi\omega\sigma R^{4}\int_{0}^{\pi}\sin^{3}\theta d\theta \quad (2)\\ &= \frac{4}{3}\pi\omega\sigma R^{4}\hat{\mathbf{z}} \quad (3)\end{aligned}$$

For $r>R$, the potential of the sphere is (Griffiths example 5.11):

$$\mathbf{A}=\frac{\mu_{0}R^{4}\omega\sigma}{3}\frac{\sin\theta}{r^{2}}\hat{\phi} \tag{4}$$

The dipole approximation for the potential is

$$\begin{aligned}\mathbf{A} &= \frac{\mu_{0}}{4\pi r^{2}}\mathbf{m}\times\hat{\mathbf{r}} \quad (5)\\ &= \frac{\mu_{0}R^{4}\omega\sigma}{3}\frac{\sin\theta}{r^{2}}\hat{\phi} \quad (6)\end{aligned}$$

which follows since θ is the angle between $\hat{\mathbf{z}}$ and $\hat{\mathbf{r}}$ so $\hat{\mathbf{z}}\times\hat{\mathbf{r}}=\sin\theta\hat{\phi}$. Thus outside the sphere, the dipole potential is the exact potential.

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