

MAGNETIC DIPOLE MOMENT OF SPINNING SPHERICAL SHELL

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.36.

Now for the magnetic dipole moment of a spinning spherical shell of charge. We can divide the sphere up into a number of circular loops. Each loop corresponds to a particular value of θ and has radius $R \sin \theta$. The current of each loop is $dI = \omega \sigma R \sin \theta d\theta$ and the magnetic moment of the loop is

$$(1) \quad d\mathbf{m} = \hat{\mathbf{z}} \pi (R \sin \theta)^2 \omega \sigma R^2 \sin \theta d\theta$$

The total dipole moment is then

$$(2) \quad \mathbf{m} = \hat{\mathbf{z}} \pi \omega \sigma R^4 \int_0^\pi \sin^3 \theta d\theta$$

$$(3) \quad = \frac{4}{3} \pi \omega \sigma R^4 \hat{\mathbf{z}}$$

For $r > R$, the potential of the sphere is (Griffiths example 5.11):

$$(4) \quad \mathbf{A} = \frac{\mu_0 R^4 \omega \sigma \sin \theta}{3} \frac{\hat{\phi}}{r^2}$$

The dipole approximation for the potential is

$$(5) \quad \mathbf{A} = \frac{\mu_0}{4\pi r^2} \mathbf{m} \times \hat{\mathbf{r}}$$

$$(6) \quad = \frac{\mu_0 R^4 \omega \sigma \sin \theta}{3} \frac{\hat{\phi}}{r^2}$$

which follows since θ is the angle between $\hat{\mathbf{z}}$ and $\hat{\mathbf{r}}$ so $\hat{\mathbf{z}} \times \hat{\mathbf{r}} = \sin \theta \hat{\phi}$. Thus outside the sphere, the dipole potential is the exact potential.

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