RADIALLY SYMMETRIC MAGNETIC FIELD

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.41.

This is a rather contrived example of the use of the Lorentz force law. We have a circular disk of radius R with a magnetic field applied perpendicular to the disk. The field depends only on the distance r from the centre of the disk, and its distribution satisfies $\int \mathbf{B}\cdot d\mathbf{a}=0$. That is, the field varies in direction and magnitude in such a way that the total flux through the disk is zero. Since the field is radially symmetric, this means

$$\int \mathbf{B}\cdot d\mathbf{a}=2\pi\int_0^R B(r)r\,dr=0 \tag{0.1}$$

Now suppose we start off a particle with charge q at the centre of the disk moving with a velocity $\mathbf{v}$. Assuming that $\mathbf{v}$ is such that the particle eventually reaches the edge of the disk, at what angle to the edge will it leave?

We can approach this problem by considering the particle's angular momentum, since if the particle leaves the disk along a line parallel to the radius vector pointing to its point of departure, the angular momentum is zero.

Since the particle starts at the origin, its initial angular momentum is zero, so the total change in angular momentum as the particle travels to the edge must be zero in order for it leave radially. The rate of change of angular momentum is the torque $\mathbf{N}$, so we have, with the total time being T:

$$\Delta\mathbf{L} = \int_0^T \mathbf{N}\,dt \tag{0.2}$$

$$= \int_0^T \mathbf{r}\times\mathbf{F}\,dt \tag{0.3}$$

$$= q\int_0^T \mathbf{r}\times(\mathbf{v}\times\mathbf{B})\,dt \tag{0.4}$$

$$= q\int_0^T \left[\mathbf{v}(\mathbf{r}\cdot\mathbf{B})-\mathbf{B}(\mathbf{r}\cdot\mathbf{v})\right]dt \tag{0.5}$$

The motion occurs entirely in the plane of the disk, so $\mathbf{r}\perp\mathbf{B}$ for the entire path, so the first term in the integrand is zero. In the second term, $\mathbf{r}\cdot\mathbf{v}=r\hat{\mathbf{r}}\cdot\mathbf{v}=r\frac{dr}{dt}$, since $\hat{\mathbf{r}}\cdot\mathbf{v}$ is the component of the velocity along the radial direction. Therefore

$$\Delta\mathbf{L} = -q\int_0^T \mathbf{B}r\frac{dr}{dt}dt \tag{0.6}$$

$$= -q\hat{\mathbf{z}}\int_0^R B(r)\,rdr \tag{0.7}$$

$$= 0 \tag{0.8}$$