MAGNETIC FORCE OF ATTRACTION BETWEEN HEMISPHERES

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.42.

Another example of the Lorentz force law. This time we'll find the magnetic force between the north and south hemispheres of a rotating shell of charge. We're faced with a bit of a dilemma here, since the magnetic field is discontinuous across the shell:

$$\mathbf{B}=\begin{cases}\frac{2\mu_0 R\omega\sigma}{3}\left(\cos\theta\hat{\mathbf{r}}-\sin\theta\hat{\theta}\right)=\frac{2\mu_0 R\omega\sigma}{3}\hat{\mathbf{z}} & r<R\\ \frac{\mu_0 R^4\omega\sigma}{3}\frac{1}{r^3}\left(2\cos\theta\hat{\mathbf{r}}+\sin\theta\hat{\theta}\right) & r>R\end{cases}\tag{1}$$

where R is the radius of the sphere, ω is the angular speed and σ is the surface charge density.

The current of an infinitesimal strip is

$$\mathbf{I}=(R\sin\theta)\,\omega\sigma\,(Rd\theta)\,\hat{\phi}\tag{2}$$

The usual procedure in cases like this is to take the average of the field on either side of the current layer. In this case, we can simplify things a little bit by noticing that the field inside the sphere is uniform and in the $\hat{\mathbf{z}}$ direction. The force on a surface element at angles θ and ϕ has the direction of $\mathbf{I}\times\mathbf{B}$, so it lies parallel to the xy plane and points radially outward (that is, there is no component in the z direction). For each surface element at a given ϕ there is an equivalent element at angle $-\phi$, so the forces on these two elements cancel. Thus the interior field exerts no net force in the z direction. The total force will therefore be half that exerted by the exterior field.

For the exterior field, we have

$$F_z=\int_0^{\pi/2}\int_0^{2\pi}\left[\mathbf{I}\times\mathbf{B}\right]_z(R\sin\theta)\,d\phi d\theta\tag{3}$$

$$=\frac{2\pi}{3}\mu_0 R^4\omega^2\sigma^2\int_0^{\pi/2}\sin\theta\left(-\sin^2\theta\hat{\mathbf{r}}-+2\sin\theta\cos\theta\hat{\theta}\right)_z d\theta\tag{4}$$

The extra factor $R\sin\theta$ in the first line comes from calculating the path length of the strip at angle θ. To get the z component of the vector in the integrand, we take the dot product with $\hat{\mathbf{z}}=\cos\theta\hat{\mathbf{r}}-\sin\theta\hat{\theta}$ and we get

$$\begin{aligned} F_z &= -\frac{2\pi}{3}\mu_0 R^4\omega^2\sigma^2\int_0^{\pi/2}3\sin^3\theta\cos\theta d\theta && (5)\\ &= -\frac{2\pi}{4}\mu_0 R^4\omega^2\sigma^2 && (6)\end{aligned}$$

To get the total force, we divide this by 2 as described above, and get, for the northern hemisphere:

$$\mathbf{F}=-\frac{\pi}{4}\mu_0 R^4\omega^2\sigma^2\hat{\mathbf{z}} \tag{7}$$

That is, the net force is attractive, since it points downward.

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