

MAGNETIC FORCE OF ATTRACTION BETWEEN HEMISPHERES

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.42.

Another example of the Lorentz force law. This time we'll find the magnetic force between the north and south hemispheres of a rotating shell of charge. We're faced with a bit of a dilemma here, since the magnetic field is discontinuous across the shell:

$$(1) \quad \mathbf{B} = \begin{cases} \frac{2\mu_0 R \omega \sigma}{3} (\cos \theta \hat{\mathbf{r}} - \sin \theta \hat{\theta}) = \frac{2\mu_0 R \omega \sigma}{3} \hat{\mathbf{z}} & r < R \\ \frac{\mu_0 R^4 \omega \sigma}{3} \frac{1}{r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\theta}) & r > R \end{cases}$$

where R is the radius of the sphere, ω is the angular speed and σ is the surface charge density.

The current of an infinitesimal strip is

$$(2) \quad \mathbf{I} = (R \sin \theta) \omega \sigma (R d\theta) \hat{\phi}$$

The usual procedure in cases like this is to take the average of the field on either side of the current layer. In this case, we can simplify things a little bit by noticing that the field inside the sphere is uniform and in the $\hat{\mathbf{z}}$ direction. The force on a surface element at angles θ and ϕ has the direction of $\mathbf{I} \times \mathbf{B}$, so it lies parallel to the xy plane and points radially outward (that is, there is no component in the z direction). For each surface element at a given ϕ there is an equivalent element at angle $-\phi$, so the forces on these two elements cancel. Thus the interior field exerts no net force in the z direction. The total force will therefore be half that exerted by the exterior field.

For the exterior field, we have

$$(3) \quad F_z = \int_0^{\pi/2} \int_0^{2\pi} [\mathbf{I} \times \mathbf{B}]_z (R \sin \theta) d\phi d\theta$$

$$(4) \quad = \frac{2\pi}{3} \mu_0 R^4 \omega^2 \sigma^2 \int_0^{\pi/2} \sin \theta (-\sin^2 \theta \hat{\mathbf{r}} - +2 \sin \theta \cos \theta \hat{\theta})_z d\theta$$

The extra factor $R \sin \theta$ in the first line comes from calculating the path length of the strip at angle θ . To get the z component of the vector in the integrand, we take the dot product with $\hat{\mathbf{z}} = \cos \theta \hat{\mathbf{r}} - \sin \theta \hat{\boldsymbol{\theta}}$ and we get

$$(5) \quad F_z = -\frac{2\pi}{3} \mu_0 R^4 \omega^2 \sigma^2 \int_0^{\pi/2} 3 \sin^3 \theta \cos \theta d\theta$$

$$(6) \quad = -\frac{2\pi}{4} \mu_0 R^4 \omega^2 \sigma^2$$

To get the total force, we divide this by 2 as described above, and get, for the northern hemisphere:

$$(7) \quad \mathbf{F} = -\frac{\pi}{4} \mu_0 R^4 \omega^2 \sigma^2 \hat{\mathbf{z}}$$

That is, the net force is attractive, since it points downward.

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