HELMHOLTZ COIL

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.46.

The magnetic field due to a circular current loop of radius R and current I a distance z above the centre of the loop is (Griffiths example 5.6):

$$B=\frac{\mu_0 I R^2}{2}\frac{1}{\left[R^2+z^2\right]^{3/2}} \tag{1}$$

If we take two such current loops a distance d apart and set $z=0$ to be halfway between them, then the net field on the axis is, by the principle of superposition:

$$B=\frac{\mu_0 I R^2}{2}\left[\frac{1}{\left[R^2+\left(z-\frac{d}{2}\right)^2\right]^{3/2}}+\frac{1}{\left[R^2+\left(z+\frac{d}{2}\right)^2\right]^{3/2}}\right] \tag{2}$$

The derivative of B is

$$\frac{\partial B}{\partial z}=\frac{\mu_0 I R^2}{2}\left(-\frac{3}{2}\frac{2z-d}{\left(R^2+\left(z-\frac{1}{2}d\right)^2\right)^{5/2}}-\frac{3}{2}\frac{2z+d}{\left(R^2+\left(z+\frac{1}{2}d\right)^2\right)^{5/2}}\right) \tag{3}$$

The derivative is always zero at $z=0$.

Now the second derivative is

$$\begin{aligned}\frac{2}{\mu_0 I R^2}\frac{\partial^2 B}{\partial z^2}=&\frac{15}{4}\frac{(2z-d)^2}{\left(R^2+\left(z-\frac{1}{2}d\right)^2\right)^{7/2}}-\frac{3}{\left(R^2+\left(z-\frac{1}{2}d\right)^2\right)^{5/2}}\\&+\frac{15}{4}\frac{(2z+d)^2}{\left(R^2+\left(z+\frac{1}{2}d\right)^2\right)^{7/2}}-\frac{3}{\left(R^2+\left(z+\frac{1}{2}d\right)^2\right)^{5/2}}\end{aligned} \tag{4}$$

At $z=0$ this becomes

$$\frac{\partial^2 B}{\partial z^2} = \frac{\mu_0 I R^2}{2}\left(\frac{15}{2}\frac{d^2}{\left(R^2+\frac{1}{4}d^2\right)^{7/2}} - \frac{6}{\left(R^2+\frac{1}{4}d^2\right)^{5/2}}\right) \tag{5}$$

We can make this derivative zero as well if we choose

$$d = R \tag{6}$$

Under this condition, the field at the centre is

$$B(0) = \frac{8I\mu_0}{5\sqrt{5}R} \tag{7}$$

Such a pair of circular loops is known as a *Helmholtz coil* and is used to produce a localized region of a relatively constant magnetic field. In fact, the third derivative also turns out to be zero at $z = 0$, so the first non-zero derivative is the fourth derivative which (if you're interested) comes out to

$$\frac{\partial^4 B}{\partial z^4}(z=0) = -\frac{27648\sqrt{5}}{3125}\frac{\mu_0 I}{R^5} \tag{8}$$