

## HELMHOLTZ COIL

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.46.

The magnetic field due to a circular current loop of radius  $R$  and current  $I$  a distance  $z$  above the centre of the loop is (Griffiths example 5.6):

$$B = \frac{\mu_0 I R^2}{2} \frac{1}{[R^2 + z^2]^{3/2}} \quad (1)$$

If we take two such current loops a distance  $d$  apart and set  $z = 0$  to be halfway between them, then the net field on the axis is, by the principle of superposition:

$$B = \frac{\mu_0 I R^2}{2} \left[ \frac{1}{[R^2 + (z - \frac{d}{2})^2]^{3/2}} + \frac{1}{[R^2 + (z + \frac{d}{2})^2]^{3/2}} \right] \quad (2)$$

The derivative of  $B$  is

$$\frac{\partial B}{\partial z} = \frac{\mu_0 I R^2}{2} \left( -\frac{3}{2} \frac{2z - d}{(R^2 + (z - \frac{1}{2}d)^2)^{5/2}} - \frac{3}{2} \frac{2z + d}{(R^2 + (z + \frac{1}{2}d)^2)^{5/2}} \right) \quad (3)$$

The derivative is always zero at  $z = 0$ .

Now the second derivative is

$$\begin{aligned} \frac{2}{\mu_0 I R^2} \frac{\partial^2 B}{\partial z^2} &= \frac{15}{4} \frac{(2z - d)^2}{(R^2 + (z - \frac{1}{2}d)^2)^{7/2}} - \frac{3}{(R^2 + (z - \frac{1}{2}d)^2)^{5/2}} \\ &+ \frac{15}{4} \frac{(2z + d)^2}{(R^2 + (z + \frac{1}{2}d)^2)^{7/2}} - \frac{3}{(R^2 + (z + \frac{1}{2}d)^2)^{5/2}} \end{aligned} \quad (4)$$

At  $z = 0$  this becomes

$$\frac{\partial^2 B}{\partial z^2} = \frac{\mu_0 I R^2}{2} \left( \frac{15}{2} \frac{d^2}{(R^2 + \frac{1}{4} d^2)^{7/2}} - \frac{6}{(R^2 + \frac{1}{4} d^2)^{5/2}} \right) \quad (5)$$

We can make this derivative zero as well if we choose

$$d = R \quad (6)$$

Under this condition, the field at the centre is

$$B(0) = \frac{8I\mu_0}{5\sqrt{5}R} \quad (7)$$

Such a pair of circular loops is known as a *Helmholtz coil* and is used to produce a localized region of a relatively constant magnetic field. In fact, the third derivative also turns out to be zero at  $z = 0$ , so the first non-zero derivative is the fourth derivative which (if you're interested) comes out to

$$\frac{\partial^4 B}{\partial z^4} (z = 0) = -\frac{27648\sqrt{5}}{3125} \frac{\mu_0 I}{R^5} \quad (8)$$