

MAGNETIC SCALAR POTENTIAL: DIPOLE AND ROTATING SPHERE

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.52.

We can try finding a magnetic scalar potential for a few examples. First, let's try a pure magnetic dipole. The field of a dipole is

$$\mathbf{B} = \frac{\mu_0 m}{4\pi r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\boldsymbol{\theta}}) \quad (1)$$

where m is the dipole moment. We can find the scalar potential by solving $\mathbf{B} = -\nabla U$ for U . In spherical coordinates

$$\nabla U = \frac{\partial U}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial U}{\partial \theta} \hat{\boldsymbol{\theta}} + \frac{1}{r \sin \theta} \frac{\partial U}{\partial \phi} \hat{\boldsymbol{\phi}} \quad (2)$$

Equating components we find that

$$\frac{\partial U}{\partial r} = -\frac{\mu_0 m}{4\pi r^3} 2 \cos \theta \quad (3)$$

$$U = \frac{\mu_0 m}{4\pi r^2} \cos \theta + f(\theta, \phi) \quad (4)$$

Checking this with the θ term, we find that

$$\frac{\partial U}{\partial \theta} = -\frac{\mu_0 m}{4\pi r^2} \sin \theta \quad (5)$$

so $f(\theta, \phi) = 0$ and

$$U = \frac{\mu_0 m}{4\pi r^2} \cos \theta \quad (6)$$

As another example, we revisit the rotating spherical shell, which had the field

$$\mathbf{B} = \begin{cases} \frac{2\mu_0 R \omega \sigma}{3} (\cos \theta \hat{\mathbf{r}} - \sin \theta \hat{\boldsymbol{\theta}}) & r < R \\ \frac{\mu_0 R^4 \omega \sigma}{3} \frac{1}{r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\boldsymbol{\theta}}) & r > R \end{cases} \quad (7)$$

The $r > R$ region is identical to the dipole, so in that region:

$$U = \frac{\mu_0 R^4 \omega \sigma}{3r^2} \cos \theta \quad (8)$$

For $r < R$, we can follow the same procedure as above and find that

$$U = -\frac{2\mu_0 R \omega \sigma}{3} r \cos \theta \quad (9)$$

Finally, we can try to find a scalar potential for the solid rotating sphere. The field inside the sphere is

$$\mathbf{B} = \frac{\mu_0 \omega Q}{4\pi R_0} \left[\left(1 - \frac{3r^2}{5R_0^2}\right) \cos \theta \hat{\mathbf{r}} + \left(\frac{6r^2}{5R_0^2} - 1\right) \sin \theta \hat{\boldsymbol{\theta}} \right] \quad (10)$$

Using the $\hat{\mathbf{r}}$ component as above, we find

$$U = -\frac{\mu_0 \omega Q}{4\pi R_0} \left(r - \frac{1}{5} \frac{r^3}{R_0^2} \right) \cos \theta + f(\theta, \phi) \quad (11)$$

where $f(\theta, \phi)$ is a constant (relative to r) of integration. From this, the θ component of the gradient must be

$$-\frac{1}{r} \frac{\partial U}{\partial \theta} = \frac{\mu_0 \omega Q}{4\pi R_0} \left(\frac{1}{5} \frac{r^2}{R_0^2} - 1 \right) \sin \theta + \frac{1}{r} \frac{\partial f}{\partial \theta} \quad (12)$$

Since f cannot depend on r , there is no way to make this agree with the θ term in \mathbf{B} above.