MAGNETIC SCALAR POTENTIAL: DIPOLE AND ROTATING SPHERE

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.52.

We can try finding a magnetic scalar potential for a few examples. First, let's try a pure magnetic dipole. The field of a dipole is

$$\mathbf{B}=\frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\hat{\mathbf{r}}+\sin\theta\hat{\theta}\right) \tag{0.1}$$

where m is the dipole moment. We can find the scalar potential by solving $\mathbf{B}=-\nabla U$ for U. In spherical coordinates

$$\nabla U=\frac{\partial U}{\partial r}\hat{\mathbf{r}}+\frac{1}{r}\frac{\partial U}{\partial\theta}\hat{\theta}+\frac{1}{r\sin\theta}\frac{\partial U}{\partial\phi}\hat{\phi} \tag{0.2}$$

Equating components we find that

$$\frac{\partial U}{\partial r} = -\frac{\mu_0 m}{4\pi r^3}2\cos\theta \tag{0.3}$$

$$U = \frac{\mu_0 m}{4\pi r^2}\cos\theta+f(\theta,\phi) \tag{0.4}$$

Checking this with the θ term, we find that

$$\frac{\partial U}{\partial\theta}=-\frac{\mu_0 m}{4\pi r^2}\sin\theta \tag{0.5}$$

so $f(\theta,\phi)=0$ and

$$U=\frac{\mu_0 m}{4\pi r^2}\cos\theta \tag{0.6}$$

As another example, we revisit the rotating spherical shell, which had the field

$$\mathbf{B}=\begin{cases}\frac{2\mu_0 R\omega\sigma}{3}\left(\cos\theta\hat{\mathbf{r}}-\sin\theta\hat{\theta}\right) & r<R\\ \frac{\mu_0 R^4\omega\sigma}{3}\frac{1}{r^3}\left(2\cos\theta\hat{\mathbf{r}}+\sin\theta\hat{\theta}\right) & r>R\end{cases} \tag{0.7}$$

The $r > R$ region is identical to the dipole, so in that region:

$$U = \frac{\mu_0 R^4 \omega\sigma}{3r^2}\cos\theta \tag{0.8}$$

For $r < R$, we can follow the same procedure as above and find that

$$U = -\frac{2\mu_0 R\omega\sigma}{3} r\cos\theta \tag{0.9}$$

Finally, we can try to find a scalar potential for the solid rotating sphere. The field inside the sphere is

$$\mathbf{B} = \frac{\mu_0 \omega Q}{4\pi R_0}\left[\left(1 - \frac{3}{5}\frac{r^2}{R_0^2}\right)\cos\theta\hat{\mathbf{r}} + \left(\frac{6}{5}\frac{r^2}{R_0^2} - 1\right)\sin\theta\hat{\boldsymbol{\theta}}\right] \tag{0.10}$$

Using the $\hat{\mathbf{r}}$ component as above, we find

$$U = -\frac{\mu_0 \omega Q}{4\pi R_0}\left(r - \frac{1}{5}\frac{r^3}{R_0^2}\right)\cos\theta + f(\theta,\phi) \tag{0.11}$$

where $f(\theta,\phi)$ is a constant (relative to r) of integration. From this, the θ component of the gradient must be

$$-\frac{1}{r}\frac{\partial U}{\partial\theta} = \frac{\mu_0 \omega Q}{4\pi R_0}\left(\frac{1}{5}\frac{r^2}{R_0^2} - 1\right)\sin\theta + \frac{1}{r}\frac{\partial f}{\partial\theta} \tag{0.12}$$

Since f cannot depend on r, there is no way to make this agree with the θ term in $\mathbf{B}$ above.