MAGNETIC SCALAR POTENTIAL: DIPOLE AND ROTATING SPHERE

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.52.

We can try finding a magnetic scalar potential for a few examples. First, let's try a pure magnetic dipole. The field of a dipole is

$$\mathbf{B}=\frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\hat{\mathbf{r}}+\sin\theta\hat{\theta}\right) \tag{1}$$

where m is the dipole moment. We can find the scalar potential by solving $\mathbf{B}=-\nabla U$ for U. In spherical coordinates

$$\nabla U=\frac{\partial U}{\partial r}\hat{\mathbf{r}}+\frac{1}{r}\frac{\partial U}{\partial\theta}\hat{\theta}+\frac{1}{r\sin\theta}\frac{\partial U}{\partial\phi}\hat{\phi} \tag{2}$$

Equating components we find that

$$\begin{aligned}\frac{\partial U}{\partial r} &= -\frac{\mu_0 m}{4\pi r^3}2\cos\theta && (3)\\ U &= \frac{\mu_0 m}{4\pi r^2}\cos\theta+f(\theta,\phi) && (4)\end{aligned}$$

Checking this with the θ term, we find that

$$\frac{\partial U}{\partial\theta}=-\frac{\mu_0 m}{4\pi r^2}\sin\theta \tag{5}$$

so $f(\theta,\phi)=0$ and

$$U=\frac{\mu_0 m}{4\pi r^2}\cos\theta \tag{6}$$

As another example, we revisit the rotating spherical shell, which had the field

$$\mathbf{B}=\begin{cases}\frac{2\mu_0 R\omega\sigma}{3}\left(\cos\theta\hat{\mathbf{r}}-\sin\theta\hat{\theta}\right) & r<R\\ \frac{\mu_0 R^4\omega\sigma}{3}\frac{1}{r^3}\left(2\cos\theta\hat{\mathbf{r}}+\sin\theta\hat{\theta}\right) & r>R\end{cases} \tag{7}$$

The $r>R$ region is identical to the dipole, so in that region:

$$U=\frac{\mu_0 R^4\omega\sigma}{3r^2}\cos\theta \tag{8}$$

For $r<R$, we can follow the same procedure as above and find that

$$U=-\frac{2\mu_0 R\omega\sigma}{3}r\cos\theta \tag{9}$$

Finally, we can try to find a scalar potential for the solid rotating sphere. The field inside the sphere is

$$\mathbf{B}=\frac{\mu_0\omega Q}{4\pi R_0}\left[\left(1-\frac{3}{5}\frac{r^2}{R_0^2}\right)\cos\theta\hat{\mathbf{r}}+\left(\frac{6}{5}\frac{r^2}{R_0^2}-1\right)\sin\theta\hat{\theta}\right] \tag{10}$$

Using the $\hat{\mathbf{r}}$ component as above, we find

$$U=-\frac{\mu_0\omega Q}{4\pi R_0}\left(r-\frac{1}{5}\frac{r^3}{R_0^2}\right)\cos\theta+f(\theta,\phi) \tag{11}$$

where $f(\theta,\phi)$ is a constant (relative to r) of integration. From this, the θ component of the gradient must be

$$-\frac{1}{r}\frac{\partial U}{\partial\theta}=\frac{\mu_0\omega Q}{4\pi R_0}\left(\frac{1}{5}\frac{r^2}{R_0^2}-1\right)\sin\theta+\frac{1}{r}\frac{\partial f}{\partial\theta} \tag{12}$$

Since f cannot depend on r, there is no way to make this agree with the θ term in $\mathbf{B}$ above.