MAGNETIC DIPOLE IN A CONSTANT FIELD

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.55.

A pure magnetic dipole has a curious property when it is placed in a uniform magnetic field: there is a sphere surrounding the dipole which is not crossed by any magnetic field lines. To see this, we start with the equation for the field of a dipole with dipole moment **m**:

$$\mathbf{B}_{dip}=\frac{\mu_0}{4\pi r^3}\left[3\left(\mathbf{m}\cdot\hat{\mathbf{r}}\right)\hat{\mathbf{r}}-\mathbf{m}\right] \tag{1}$$

We'll align the dipole so that it points along the negative z axis: $\mathbf{m}=-m_0\hat{\mathbf{z}}$ and apply the magnetic field also along the z axis: $\mathbf{B}_{ext}=B_0\hat{\mathbf{z}}$. Then the total field is

$$\begin{aligned}\mathbf{B} &= \mathbf{B}_{dip}+\mathbf{B}_{ext} &(2)\\ &= \frac{\mu_0}{4\pi r^3}\left[-3m_0\cos\theta\hat{\mathbf{r}}+m_0\hat{\mathbf{z}}\right]+B_0\hat{\mathbf{z}} &(3)\end{aligned}$$

If there is a sphere which is not crossed by field lines, then **B** must be tangent to the sphere everywhere on that sphere. In other words, **B** is perpendicular to the surface normal vector, which for a sphere is just $\hat{\mathbf{r}}$. So we look for a solution of $\mathbf{B}\cdot\hat{\mathbf{r}}=0$. We get

$$\begin{aligned}\mathbf{B}\cdot\hat{\mathbf{r}} &= \frac{\mu_0}{4\pi r^3}\left[-3m_0\cos\theta+m_0\cos\theta\right]+B_0\cos\theta=0 &(4)\\ \frac{m_0\mu_0}{2\pi r^3} &= B_0 &(5)\\ r &= \left(\frac{m_0\mu_0}{2\pi B_0}\right)^{1/3} &(6)\end{aligned}$$

Presumably the field lines inside this sphere resemble those of the pure dipole; that is, they emerge outwards from the north pole of the dipole and loop round to converge on the south pole. Outside the sphere the field lines far from the sphere are straight lines, while closer to the sphere they diverge around the sphere, just like water flowing around an obstacle in a stream.