

GYROMAGNETIC RATIO

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.56.

The *gyromagnetic ratio* γ is the ratio of an object's magnetic dipole moment to its angular momentum. The numerical value of γ depends on the units being used (that is, it's not a dimensionless quantity).

For a circular wire loop of radius r carrying charge Q and having mass M we can calculate γ as follows. Suppose the loop is rotating with angular speed ω . Then the current in the loop is $I = \frac{Q}{2\pi r} r \omega = \frac{Q}{2\pi} \omega$. The magnetic dipole moment is therefore

$$(1) \quad m = Ia = \frac{Q}{2\pi} \omega \pi r^2 = \frac{Q}{2} \omega r^2$$

The angular momentum is $L = mr^2\omega$, so the gyromagnetic ratio of a circular wire loop is

$$(2) \quad \gamma = \frac{m}{L} = \frac{Q}{2M}$$

The ratio is independent of r and ω , which might seem a bit bizarre since we can calculate γ even when the loop isn't spinning, and therefore there is no current or angular momentum. In that case, γ should be interpreted as a limit, since otherwise it would involve dividing zero by zero.

Since the ratio is independent of r , it applies to any circular loop so we can apply this to any solid of revolution, such as a sphere or cylinder. All such objects have the same formula for γ .

We can attempt to find γ for an elementary particle such as an electron, although the resulting answer isn't correct (as you might expect when applying classical physics to quantum objects). The electron has spin $\frac{1}{2}$ which means its angular momentum is $\frac{\hbar}{2}$. We then get

$$(3) \quad \gamma_e = \frac{2m_e}{\hbar} = -\frac{e}{2M_e}$$

where m_e is the dipole moment of the electron, M_e is its mass and e is the elementary charge. Plugging in the numbers we get (in SI units):

$$(4) \quad \gamma_e = -\frac{1.602 \times 10^{-19}}{2 \times 9.11 \times 10^{-31}} = -8.792 \times 10^{10} \text{C} \cdot \text{kg}^{-1}$$

From this we can get the magnetic dipole moment of the electron

$$(5) \quad m_e = \frac{\hbar}{2} \gamma_e = -4.636 \times 10^{-24} \text{Amp} \cdot \text{m}^2$$

Experimentally, the value is $m_e = -9.285 \times 10^{-24} \text{Amp} \cdot \text{m}^2$ which is almost exactly twice the calculated value. When the moment is calculated using quantum electrodynamics, the agreement is pretty much exact.

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