# TORQUE ON A MAGNETIC DIPOLE

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 6.1.

As a prelude to looking at magnetic fields in matter, we need to work out the torque and force on a magnetic dipole, since atoms are composed of such dipoles in the form of electrons (which have both orbital and spin magnetic moments).

Consider a small square current loop carrying current $I$ and having side length $a$. Place the loop initially in the $xy$ plane with its centre at the origin and sides aligned with the coordinate axes, and then apply a constant magnetic field $\mathbf{B}$ in the $z$ direction. Now if we rotate the loop about the $x$ axis by an angle $\theta$, what are the net force and torque due to the field $\mathbf{B}$?

Using the Lorentz force law, the force on each side of the square is $\mathbf{F}=a\mathbf{I}\times\mathbf{B}$ and, by using the right hand rule, we see that the forces on the two sides parallel to the $y$ axis point in the $\pm x$ directions and thus cancel, and the forces on the sides parallel to the $x$ axis point in the $\pm y$ directions and also cancel, so that the net force on the loop is zero (although the forces all tend to stretch the loop by pulling it outwards on all four sides).

The torque on the sides parallel to the $y$ axis is zero, but on the other two sides, the torques do not cancel. To see this, recall that the definition of the torque is $\mathbf{N}=\mathbf{r}\times\mathbf{F}$. For a segment on one of the sides, we can split $\mathbf{r}=\mathbf{r}_{\parallel}+\mathbf{r}_{\perp}$ where $\mathbf{r}_{\parallel}$ is the component of $\mathbf{r}$ that is parallel to the side of the loop and $\mathbf{r}_{\perp}$ is perpendicular to the side. On a given edge, each contribution of $\mathbf{r}_{\parallel}\times\mathbf{F}$ from a small segment on one side of the centre of this edge is cancelled by an equal and opposite contribution from a small segment on the other side of the centre, so the net torque is the sum of $\mathbf{r}_{\perp}\times\mathbf{F}$ from all the segments on the edge, where $\mathbf{F}=\mathbf{I}\times\mathbf{B}dx$ is the force on the small segment. Now $|\mathbf{r}_{\perp}|=\frac{a}{2}$ so the torque on one edge is $N=\int_{-a/2}^{a/2}\frac{a}{2}IB\sin\theta dx=\frac{a^2}{2}IB\sin\theta$. If $\mathbf{I}$ on this edge points in the $x$ direction, the direction of $\mathbf{N}$ is also $x$. There will be an equal torque from the opposite edge, since both $\mathbf{r}_{\perp}$ and $\mathbf{I}$ point in opposite directions on the opposite edge, so the total torque is

$$\mathbf{N}=2\frac{a^2}{2}IB\sin\theta\hat{\mathbf{x}}=a^2IB\sin\theta\hat{\mathbf{x}}=\mathbf{m}\times\mathbf{B} \tag{1}$$

where $\mathbf{m}=I\mathbf{a}$ is the magnetic moment of the loop.

As a simple example, suppose we have a small circular current loop of radius $b$ lying in the $xy$ plane at the origin, and a square current loop with side length $a$ on the $y$ axis at a distance $r$ from the origin. Both loops carry current $I$.

If we take $r\gg a,b$, then we can approximate the circular loop as a pure dipole with a magnetic moment of

$$\mathbf{m}=\pi Ib^{2}\hat{\mathbf{z}} \tag{2}$$

The magnetic field due to a dipole is

$$\mathbf{B}=\frac{\mu_{0}}{4\pi r^{3}}\left[3\left(\mathbf{m}\cdot\hat{\mathbf{r}}\right)\hat{\mathbf{r}}-\mathbf{m}\right] \tag{3}$$

In this case, $\hat{\mathbf{r}}=\hat{\mathbf{y}}$ so

$$\mathbf{B}=-\frac{\mu_{0}}{4\pi r^{3}}\pi Ib^{2}\hat{\mathbf{z}}=-\frac{Ib^{2}\mu_{0}}{4r^{3}}\hat{\mathbf{z}} \tag{4}$$

The torque on the square loop is then

$$\mathbf{N}=-\frac{\mu_{0}I^{2}a^{2}b^{2}}{4r^{3}}\hat{\mathbf{m}}\times\hat{\mathbf{z}} \tag{5}$$

where $\hat{\mathbf{m}}$ is a unit vector in the direction of the square's magnetic moment. If the square is free to rotate, it will line itself up so that $\hat{\mathbf{m}}=-\hat{\mathbf{z}}$.

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