

FORCE BETWEEN TWO MAGNETIC DIPOLES

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References: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 6.3.

A general formula for the force on a magnetic dipole due to a varying magnetic field is

$$(0.1) \quad \mathbf{F} = \nabla (\mathbf{m} \cdot \mathbf{B})$$

A special case of this occurs when we have a circular current loop suspended in a radially symmetric field (such as that produced at the end of a solenoid, or along the axis of another dipole). In that case, imagine that at the location of the circular loop, the field spreads outwards like the petals of a flower, so that it has a vertical component (parallel to the axis of symmetry) and a horizontal, radial component, perpendicular to the axis. The force due to the field on the current is

$$(0.2) \quad \mathbf{F} = I \int d\ell \times \mathbf{B}$$

integrated around the circle. The contributions from the vertical component of \mathbf{B} cancel out, but the horizontal component of magnitude $B \cos \alpha$ (where the angle between the field and the horizontal is α) adds up around the circle to give a net force (downwards) of

$$(0.3) \quad F = 2\pi RIB \cos \alpha$$

where R is the radius of the circular loop.

We can apply both these formulas to the case of two magnetic dipoles \mathbf{m}_1 and \mathbf{m}_2 oriented along the z axis (so their axes are parallel) and separated by a distance r . First, we use the gradient formula at the top. We start with the dipole field of \mathbf{m}_1 in spherical coordinates:

$$(0.4) \quad \mathbf{B} = \frac{\mu_0 m_1}{4\pi r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\boldsymbol{\theta}})$$

Since $\mathbf{m}_2 = m_2 \hat{\mathbf{z}}$, we get

$$(0.5) \quad \mathbf{m}_2 \cdot \mathbf{B} = \frac{\mu_0 m_1 m_2}{4\pi r^3} (2 \cos^2 \theta - \sin^2 \theta)$$

Taking the gradient in spherical coordinates, we get

$$(0.6) \quad \mathbf{F} = \nabla (\mathbf{m}_2 \cdot \mathbf{B})$$

$$(0.7) \quad = \frac{\mu_0 m_1 m_2}{4\pi r^4} [-3 (2 \cos^2 \theta - \sin^2 \theta) \hat{\mathbf{r}} - 6 \sin \theta \cos \theta \hat{\boldsymbol{\theta}}]$$

In the case of an infinitesimal dipole on the z axis, $\theta = 0$ and we get

$$(0.8) \quad \mathbf{F} = -\frac{3\mu_0 m_1 m_2}{2\pi r^4} \hat{\mathbf{r}}$$

$$(0.9) \quad = -\frac{3\mu_0 m_1 m_2}{2\pi r^4} \hat{\mathbf{z}}$$

That is, the dipole \mathbf{m}_2 feels an attractive force towards \mathbf{m}_1 .

Doing the same calculation with the seemingly simpler formula 0.3 is actually more complicated, mainly because we're dealing with an ideal, and thus infinitesimal, dipole rather than a finite circular current loop. We can start by working out the field due to \mathbf{m}_1 at the location of a finite loop with radius R . Here, we'll redefine r as the distance from \mathbf{m}_1 to the centre of the loop, so the field on the loop itself is then

$$(0.10) \quad \mathbf{B} = \frac{\mu_0 m_1}{4\pi \rho^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\boldsymbol{\theta}})$$

where

$$(0.11) \quad \rho \equiv \sqrt{r^2 + R^2}$$

$$(0.12) \quad \cos \theta = \frac{r}{\rho}$$

$$(0.13) \quad \sin \theta = \frac{R}{\rho}$$

We need the horizontal component of \mathbf{B} at the loop, which we can get by considering the r and θ components separately. The horizontal component of B_r is

$$(0.14) \quad B_{rh} = \frac{\mu_0 m_1}{4\pi \rho^3} \left(2 \cos \theta \cos \left(\frac{\pi}{2} - \theta \right) \right)$$

$$(0.15) \quad = \frac{\mu_0 m_1}{4\pi \rho^3} (2 \cos \theta \sin \theta)$$

Similarly, the horizontal component of B_θ is

$$(0.16) \quad B_{\theta h} = \frac{\mu_0 m_1}{4\pi\rho^3} (\sin\theta \cos\theta)$$

The total horizontal component of the field is the sum of these two, and replaces the $B \cos\alpha$ factor in 0.3:

$$(0.17) \quad B_h = \frac{3\mu_0 m_1}{4\pi\rho^3} \sin\theta \cos\theta$$

Now we need to express the $2\pi RI$ factor in terms of m_2 . The magnetic moment of the loop is $m_2 = \pi R^2 I$, so

$$(0.18) \quad 2\pi RI = \frac{2m_2}{R}$$

and the force on the loop is then

$$(0.19) \quad F = \frac{3\mu_0 m_1 m_2}{2\pi\rho^3 R} \sin\theta \cos\theta$$

To get the formula for an infinitesimal loop, we need to take the limit as $R \rightarrow 0$. In this limit $\rho \rightarrow r$, $\cos\theta \rightarrow 1$ and $\sin\theta \rightarrow R/r$ so we get

$$(0.20) \quad \lim_{R \rightarrow 0} F = \frac{3\mu_0 m_1 m_2}{2\pi r^4}$$

which agrees (in magnitude) with the formula obtained above by the gradient method.

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