# FORCE ON A MAGNETIC DIPOLE IN A SLAB OF CURRENT

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References: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 6.5.

A general formula for the force on a magnetic dipole due to a varying magnetic field is

$$\mathbf{F}=\nabla(\mathbf{m}\cdot\mathbf{B}) \tag{1}$$

We can apply this formula to a case we've considered before: a slab of uniform current. The slab is parallel to the $yz$ plane and extends from $x=-a$ to $x=+a$. The current density is $\mathbf{J}=J_0\hat{\mathbf{z}}$. Using our earlier result, the magnetic field inside the slab is

$$\mathbf{B}=\mu_0 J_0 x\hat{\mathbf{y}} \tag{2}$$

where the direction is determined by the right-hand rule.

For a dipole $\mathbf{m}=m_0\hat{\mathbf{x}}$ at the origin, the force is zero, since $\mathbf{m}\cdot\mathbf{B}=0$. For $\mathbf{m}=m_0\hat{\mathbf{y}}$, $\mathbf{m}\cdot\mathbf{B}=\mu_0 m_0 J_0 x$, so the force is

$$\mathbf{F}=\mu_0 m_0 J_0\hat{\mathbf{x}} \tag{3}$$

Note that even though $\mathbf{m}$ is a constant, we can't take it outside the $\nabla$ operator. For example, with $\mathbf{m}=m_0\hat{\mathbf{x}}$, we would get

$$(\mathbf{m}\cdot\nabla)\,\mathbf{B}=\mu_0 m_0 J_0\hat{\mathbf{y}}\neq 0 \tag{4}$$

and for $\mathbf{m}=m_0\hat{\mathbf{y}}$ we would get

$$(\mathbf{m}\cdot\nabla)\,\mathbf{B}=0\neq\mu_0 m_0 J_0\hat{\mathbf{x}} \tag{5}$$

In the electrostatic analog, $\mathbf{F}=(\mathbf{p}\cdot\nabla)\,\mathbf{E}=\nabla(\mathbf{p}\cdot\mathbf{E})$. If we expand the second term using the vector product rule, we get

$$\nabla(\mathbf{p}\cdot\mathbf{E})=\mathbf{p}\times(\nabla\times\mathbf{E})+\mathbf{E}\times(\nabla\times\mathbf{p})+(\mathbf{p}\cdot\nabla)\,\mathbf{E}+(\mathbf{E}\cdot\nabla)\,\mathbf{p} \tag{6}$$

If $\mathbf{p}$ is a constant, the second and fourth terms are zero, and because $\nabla\times\mathbf{E}=0$ in electrostatics, the first term is zero, which proves the relation. In the magnetic case, $\nabla\times\mathbf{B}=\mu_0\mathbf{J}\neq 0$ so

$$\nabla(\mathbf{m}\cdot\mathbf{B})=\mu_0\mathbf{m}\times\mathbf{J}+(\mathbf{m}\cdot\nabla)\mathbf{B} \tag{7}$$

We can see that by adding the $\mu_0\mathbf{m}\times\mathbf{J}$ term to the $(\mathbf{m}\cdot\nabla)\mathbf{B}$ terms calculated above, we do indeed get the correct values as calculated by 1.