

## FORCE ON A MAGNETIC DIPOLE IN A SLAB OF CURRENT

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References: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 6.5.

A general formula for the force on a magnetic dipole due to a varying magnetic field is

$$\mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B}) \quad (1)$$

We can apply this formula to a case we've considered before: a slab of uniform current. The slab is parallel to the  $yz$  plane and extends from  $x = -a$  to  $x = +a$ . The current density is  $\mathbf{J} = J_0\hat{\mathbf{z}}$ . Using our earlier result, the magnetic field inside the slab is

$$\mathbf{B} = \mu_0 J_0 x \hat{\mathbf{y}} \quad (2)$$

where the direction is determined by the right-hand rule.

For a dipole  $\mathbf{m} = m_0\hat{\mathbf{x}}$  at the origin, the force is zero, since  $\mathbf{m} \cdot \mathbf{B} = 0$ . For  $\mathbf{m} = m_0\hat{\mathbf{y}}$ ,  $\mathbf{m} \cdot \mathbf{B} = \mu_0 m_0 J_0 x$ , so the force is

$$\mathbf{F} = \mu_0 m_0 J_0 \hat{\mathbf{x}} \quad (3)$$

Note that even though  $\mathbf{m}$  is a constant, we can't take it outside the  $\nabla$  operator. For example, with  $\mathbf{m} = m_0\hat{\mathbf{x}}$ , we would get

$$(\mathbf{m} \cdot \nabla) \mathbf{B} = \mu_0 m_0 J_0 \hat{\mathbf{y}} \neq 0 \quad (4)$$

and for  $\mathbf{m} = m_0\hat{\mathbf{y}}$  we would get

$$(\mathbf{m} \cdot \nabla) \mathbf{B} = 0 \neq \mu_0 m_0 J_0 \hat{\mathbf{x}} \quad (5)$$

In the electrostatic analog,  $\mathbf{F} = (\mathbf{p} \cdot \nabla) \mathbf{E} = \nabla(\mathbf{p} \cdot \mathbf{E})$ . If we expand the second term using the vector product rule, we get

$$\nabla(\mathbf{p} \cdot \mathbf{E}) = \mathbf{p} \times (\nabla \times \mathbf{E}) + \mathbf{E} \times (\nabla \times \mathbf{p}) + (\mathbf{p} \cdot \nabla) \mathbf{E} + (\mathbf{E} \cdot \nabla) \mathbf{p} \quad (6)$$

If  $\mathbf{p}$  is a constant, the second and fourth terms are zero, and because  $\nabla \times \mathbf{E} = 0$  in electrostatics, the first term is zero, which proves the relation. In the magnetic case,  $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \neq 0$  so

$$\nabla(\mathbf{m} \cdot \mathbf{B}) = \mu_0 \mathbf{m} \times \mathbf{J} + (\mathbf{m} \cdot \nabla) \mathbf{B} \quad (7)$$

We can see that by adding the  $\mu_0 \mathbf{m} \times \mathbf{J}$  term to the  $(\mathbf{m} \cdot \nabla) \mathbf{B}$  terms calculated above, we do indeed get the correct values as calculated by 1.