

MAGNETIC FIELD WITHIN A CAVITY

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References: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 6.13.

We can do a similar analysis to that for cavities in a dielectric to calculate the magnetic fields in cavities within magnetized material. We start with a magnetic material within which the magnetic field is \mathbf{B}_0 and the magnetization is \mathbf{M} (neither of which need be constant). The auxiliary field is $\mathbf{H}_0 = \frac{1}{\mu_0}\mathbf{B}_0 - \mathbf{M}$.

If we hollow out a spherical cavity we can simulate this by superimposing a sphere with opposite magnetization onto the material. Griffiths shows in his example 6.1 that the internal magnetic field due to a uniformly magnetized sphere with magnetization $-\mathbf{M}$ is $\mathbf{B}_s = -\frac{2}{3}\mu_0\mathbf{M}$ so the net field inside the cavity is

$$(0.1) \quad \mathbf{B} = \mathbf{B}_0 - \frac{2}{3}\mu_0\mathbf{M}$$

Since the magnetization within the cavity is zero, the auxiliary field is

$$(0.2) \quad \mathbf{H} = \frac{1}{\mu_0}\mathbf{B} = \frac{1}{\mu_0}\mathbf{B}_0 - \frac{2}{3}\mathbf{M} = \mathbf{H}_0 + \frac{1}{3}\mathbf{M}$$

For a needle-shaped cavity with magnetization $-\mathbf{M}$, a bound surface current is induced which is

$$(0.3) \quad \mathbf{K}_b = -\mathbf{M} \times \hat{\mathbf{n}} = -M\hat{\phi}$$

This is effectively a solenoid, so its field within the needle is $\mathbf{B}_s = -\mu_0\mathbf{M}$ and the net field is

$$(0.4) \quad \mathbf{B} = \mathbf{B}_0 - \mu_0\mathbf{M}$$

$$(0.5) \quad \mathbf{H} = \frac{1}{\mu_0}\mathbf{B}_0 - \mathbf{M} = \mathbf{H}_0$$

Finally, for a thin circular wafer of radius R and thickness t , the surface current is the same as for the needle, but the setup can be approximated

by a circular current loop, the field of which is derived by Griffiths in his example 5.6. For a point in the centre of the loop, this gives

$$(0.6) \quad B = \frac{\mu_0 I}{2R}$$

where I is the current in the loop. In our case, $I = -Mt$, so the field is

$$(0.7) \quad \mathbf{B}_w = -\frac{\mu_0 t}{2R} \mathbf{M}$$

If $t \ll R$ (that is, the wafer is very thin), then $\mathbf{B}_w \approx 0$ so

$$(0.8) \quad \mathbf{B} = \mathbf{B}_0$$

$$(0.9) \quad \mathbf{H} = \frac{1}{\mu_0} \mathbf{B}_0 = \mathbf{H}_0 + \mathbf{M}$$

Thus \mathbf{B} is unchanged for the wafer and \mathbf{H} is unchanged for the needle.

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