References: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 6.16.

The magnetic analog to the electric dielectric constant is the magnetic permeability. In a linear magnetic material, the magnetization is directly proportional to the auxiliary field, so we have

$$\mathbf{M} = \chi_m \mathbf{H} \tag{0.1}$$

where χ_m is the *magnetic susceptibility*. Given this proportionality, the field is also directly proportional to $\mathbf{H}$:

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) = \mu_0 (1 + \chi_m) \mathbf{H} \equiv \mu \mathbf{H} \tag{0.2}$$

where μ is the magnetic permeability. In a vacuum, the magnetization is always zero, so $\chi_m = 0$ in this case, and the permeability becomes $\mu = \mu_0$, which is known as the permeability of free space.

The permeability is a property of the material so different substances have different permeabilities. In general, $\mathbf{H}$ can be found without knowing anything about the material, but in order for $\mathbf{B}$ to be found, the permeability must be known.

As a simple example, suppose we have two infinitely long concentric tubes (hollow cylinders), with the inner tube having radius a and carrying current $I\hat{\mathbf{z}}$ and the outer tube having radius b and current $-I\hat{\mathbf{z}}$.

By symmetry, we know that $\mathbf{B}$, $\mathbf{H}$ and $\mathbf{M}$ are all circumferential (that is, proportional to $\hat{\phi}$) so we can use Ampère's law:

$$\oint \mathbf{H} \cdot d\ell = I_f \tag{0.3}$$

where I_f is the free current enclosed by the path of integration. Choosing a circular path of radius r between the two cylinders we get

$$2\pi r H = I \tag{0.4}$$

$$\mathbf{H} = \frac{I}{2\pi r}\hat{\phi} \tag{0.5}$$

$$\mathbf{B} = \mu_0 (1 + \chi_m) \mathbf{H} \tag{0.6}$$

$$= \mu_0 (1 + \chi_m) \frac{I}{2\pi r}\hat{\phi} \tag{0.7}$$

We can also find the magnetization and resulting bound currents. We have

$$\mathbf{M} = \chi_m \mathbf{H} = \frac{I\chi_m}{2\pi r}\hat{\phi} \tag{0.8}$$

$$\mathbf{J}_b = \nabla \times \mathbf{M} = 0 \tag{0.9}$$

$$\mathbf{K}_{bi} = \mathbf{M} \times \hat{\mathbf{n}} = \frac{I\chi_m}{2\pi a}\hat{\mathbf{z}} \tag{0.10}$$

where $\mathbf{K}_{bi}$ in the last line is the bound surface current density on the inner cylinder, where the normal vector points inwards since we're in the region between the cylinders, so $\hat{\mathbf{n}} = -\hat{\mathbf{r}}$. Having determined the bound currents, we can work out $\mathbf{B}$ from these currents (and the free currents) without worrying about the magnetization. Since the field due to a wire carrying a total current I_t is (from Ampère's law):

$$\mathbf{B} = \frac{\mu_0 I_t}{2\pi r}\hat{\phi} \tag{0.11}$$

and the total bound current flowing on the inner cylinder is $2\pi a K_{bi} = I\chi_m$ the field due to the combined free and bound currents here is

$$\mathbf{B} = \frac{\mu_0 (1+\chi_m) I}{2\pi r}\hat{\phi} \tag{0.12}$$

in agreement with the earlier calculation. Note that the outer cylinder has no effect on the field between the cylinders since the path of integration excludes this current. If we go outside the outer cylinder, the net free current is zero so we get

$$2\pi r H = 0 \tag{0.13}$$

$$H = 0 \tag{0.14}$$

so $\mathbf{B} = 0$ here as well. The bound current on the outer cylinder is

$$\mathbf{K}_{bo} = \mathbf{M} \times \hat{\mathbf{n}} = -\frac{I\chi_m}{2\pi b}\hat{\mathbf{z}} \tag{0.15}$$

since here the normal to the surface points outwards, so $\hat{\mathbf{n}} = +\hat{\mathbf{r}}$. The total bound current on the outer cylinder is thus $2\pi K_{bo} = -I\chi_m$ so the total bound current from both cylinders is zero, making the total current zero.

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