MAGNETIC FIELD WITHIN A CURRENT-CARRYING WIRE

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References: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 6.17.

Another example of using the auxiliary field to find the magnetic field. This time, we have a long wire of radius a carrying a uniform current I. The wire is made of a linear metal so that the magnetization is directly proportional to $\mathbf{H}$.

$$\mathbf{M}=\chi_m\mathbf{H} \tag{0.1}$$

where χ_m is the *magnetic susceptibility*. Given this proportionality, the field is also directly proportional to $\mathbf{H}$:

$$\mathbf{B}=\mu_0(\mathbf{H}+\mathbf{M})=\mu_0(1+\chi_m)\mathbf{H}\equiv\mu\mathbf{H} \tag{0.2}$$

where μ is the magnetic permeability.

From the symmetry of the setup, $\mathbf{H}$ is circumferential, so we can take a circular integration path to get

$$\oint\mathbf{H}\cdot d\ell = I_f \tag{0.3}$$

$$2\pi rH = \frac{r^2}{a^2}I \tag{0.4}$$

$$\mathbf{H} = \frac{rI}{2\pi a^2}\hat{\phi} \tag{0.5}$$

From this we get the field for $r<a$

$$\mathbf{B}=\frac{\mu_0(1+\chi_m)rI}{2\pi a^2}\hat{\phi} \tag{0.6}$$

Outside the wire, $\chi_m=0$ and the enclosed free current is just I, so for $r>a$

$$\mathbf{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi} \tag{0.7}$$

The magnetization is

$$\mathbf{M} = \frac{\chi_m r I}{2\pi a^2}\hat{\phi} \tag{0.8}$$

so the bound currents are

$$\mathbf{J}_b = \nabla \times \mathbf{M} \tag{0.9}$$

$$= \frac{\chi_m I}{\pi a^2}\hat{\mathbf{z}} \tag{0.10}$$

$$\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}} \tag{0.11}$$

$$= -\frac{\chi_m I}{2\pi a}\hat{\mathbf{z}} \tag{0.12}$$

The total bound current is

$$\mathbf{I}_b = \pi a^2 \mathbf{J}_b + 2\pi a \mathbf{K}_b = 0 \tag{0.13}$$