

MAGNETIC FIELD WITHIN A CURRENT-CARRYING WIRE

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References: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 6.17.

Another example of using the auxiliary field to find the magnetic field. This time, we have a long wire of radius a carrying a uniform current I . The wire is made of a linear metal so that the magnetization is directly proportional to \mathbf{H} .

$$\mathbf{M} = \chi_m \mathbf{H} \quad (1)$$

where χ_m is the *magnetic susceptibility*. Given this proportionality, the field is also directly proportional to \mathbf{H} :

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) = \mu_0 (1 + \chi_m) \mathbf{H} \equiv \mu \mathbf{H} \quad (2)$$

where μ is the magnetic permeability.

From the symmetry of the setup, \mathbf{H} is circumferential, so we can take a circular integration path to get

$$\oint \mathbf{H} \cdot d\ell = I_f \quad (3)$$

$$2\pi r H = \frac{r^2}{a^2} I \quad (4)$$

$$\mathbf{H} = \frac{rI}{2\pi a^2} \hat{\phi} \quad (5)$$

From this we get the field for $r < a$

$$\mathbf{B} = \frac{\mu_0 (1 + \chi_m) r I}{2\pi a^2} \hat{\phi} \quad (6)$$

Outside the wire, $\chi_m = 0$ and the enclosed free current is just I , so for $r > a$

$$\mathbf{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi} \quad (7)$$

The magnetization is

$$\mathbf{M} = \frac{\chi_m r I}{2\pi a^2} \hat{\phi} \quad (8)$$

so the bound currents are

$$\mathbf{J}_b = \nabla \times \mathbf{M} \quad (9)$$

$$= \frac{\chi_m I}{\pi a^2} \hat{\mathbf{z}} \quad (10)$$

$$\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}} \quad (11)$$

$$= -\frac{\chi_m I}{2\pi a} \hat{\mathbf{z}} \quad (12)$$

The total bound current is

$$\mathbf{I}_b = \pi a^2 \mathbf{J}_b + 2\pi a \mathbf{K}_b = 0 \quad (13)$$