

MAGNETIC FIELD WITHIN A CURRENT-CARRYING WIRE

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References: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 6.17.

Another example of using the auxiliary field to find the magnetic field. This time, we have a long wire of radius a carrying a uniform current I . The wire is made of a linear metal so that the magnetization is directly proportional to \mathbf{H} .

$$(1) \quad \mathbf{M} = \chi_m \mathbf{H}$$

where χ_m is the *magnetic susceptibility*. Given this proportionality, the field is also directly proportional to \mathbf{H} :

$$(2) \quad \mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) = \mu_0 (1 + \chi_m) \mathbf{H} \equiv \mu \mathbf{H}$$

where μ is the magnetic permeability.

From the symmetry of the setup, \mathbf{H} is circumferential, so we can take a circular integration path to get

$$(3) \quad \oint \mathbf{H} \cdot d\ell = I_f$$

$$(4) \quad 2\pi r H = \frac{r^2}{a^2} I$$

$$(5) \quad \mathbf{H} = \frac{rI}{2\pi a^2} \hat{\phi}$$

From this we get the field for $r < a$

$$(6) \quad \mathbf{B} = \frac{\mu_0 (1 + \chi_m) r I}{2\pi a^2} \hat{\phi}$$

Outside the wire, $\chi_m = 0$ and the enclosed free current is just I , so for $r > a$

$$(7) \quad \mathbf{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi}$$

The magnetization is

$$(8) \quad \mathbf{M} = \frac{\chi_m r I}{2\pi a^2} \hat{\phi}$$

so the bound currents are

$$(9) \quad \mathbf{J}_b = \nabla \times \mathbf{M}$$

$$(10) \quad = \frac{\chi_m I}{\pi a^2} \hat{z}$$

$$(11) \quad \mathbf{K}_b = \mathbf{M} \times \hat{n}$$

$$(12) \quad = -\frac{\chi_m I}{2\pi a} \hat{z}$$

The total bound current is

$$(13) \quad \mathbf{I}_b = \pi a^2 \mathbf{J}_b + 2\pi a \mathbf{K}_b = 0$$