MAGNETIC FIELD OF A SPHERE IN A UNIFORM FIELD

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 6.18.

If we place a linear magnetic sphere in a uniform magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$, we can find the magnetic field inside the sphere using the method of successive approximations we used for a dielectric sphere in an electric field.

We start by assuming that the only field present is the uniform field, which then induces a magnetization within the sphere:

$$\mathbf{M}_0 = \frac{\chi_m}{\mu}\mathbf{B}_0 \tag{1}$$

We then find the field produced by this magnetization (which is done by Griffiths in example 6.1):

$$\mathbf{B}_1 = \frac{2}{3}\mu_0\mathbf{M}_0 = \frac{2\mu_0\chi_m}{3\mu}\mathbf{B}_0 \tag{2}$$

This field produces another bit of magnetization:

$$\mathbf{M}_1 = \frac{\chi_m}{\mu}\mathbf{B}_1 = \frac{2\mu_0\chi_m^2}{3\mu^2}\mathbf{B}_0 \tag{3}$$

which in turn produces a bit more field:

$$\mathbf{B}_2 = \frac{2}{3}\mu_0\mathbf{M}_1 = \left(\frac{2\mu_0\chi_m}{3\mu}\right)^2\mathbf{B}_0 \tag{4}$$

The process repeats so that for the nth iteration we get

$$\mathbf{B}_n = \left(\frac{2\mu_0\chi_m}{3\mu}\right)^n\mathbf{B}_0 \tag{5}$$

The total field is then the sum of all the individual contributions:

$$\begin{aligned}
\mathbf{B} &= \mathbf{B}_0\sum_{n=0}^{\infty}\left(\frac{2\mu_0\chi_m}{3\mu}\right)^n && (6)\\
&= \mathbf{B}_0\frac{1}{1-\frac{2\mu_0\chi_m}{3\mu}} && (7)\\
&= \mathbf{B}_0\frac{1}{1-\frac{2\chi_m}{3(1+\chi_m)}} && (8)\\
&= \mathbf{B}_0\frac{3+3\chi_m}{3+\chi_m} && (9)
\end{aligned}$$

Unfortunately, the same method doesn't work for finding the field outside the sphere, since that field is not constant in space.

PINGBACKS

Pingback: Magnetostatic formulas from electrostatic formulas