MAGNETIC FIELD OF A SPHERE IN A UNIFORM FIELD

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 6.18.

If we place a linear magnetic sphere in a uniform magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$, we can find the magnetic field inside the sphere using the method of successive approximations we used for a dielectric sphere in an electric field.

We start by assuming that the only field present is the uniform field, which then induces a magnetization within the sphere:

$$\mathbf{M}_0 = \frac{\chi_m}{\mu}\mathbf{B}_0 \tag{0.1}$$

We then find the field produced by this magnetization (which is done by Griffiths in example 6.1):

$$\mathbf{B}_1 = \frac{2}{3}\mu_0\mathbf{M}_0 = \frac{2\mu_0\chi_m}{3\mu}\mathbf{B}_0 \tag{0.2}$$

This field produces another bit of magnetization:

$$\mathbf{M}_1 = \frac{\chi_m}{\mu}\mathbf{B}_1 = \frac{2\mu_0\chi_m^2}{3\mu^2}\mathbf{B}_0 \tag{0.3}$$

which in turn produces a bit more field:

$$\mathbf{B}_2 = \frac{2}{3}\mu_0\mathbf{M}_1 = \left(\frac{2\mu_0\chi_m}{3\mu}\right)^2\mathbf{B}_0 \tag{0.4}$$

The process repeats so that for the nth iteration we get

$$\mathbf{B}_n = \left(\frac{2\mu_0\chi_m}{3\mu}\right)^n\mathbf{B}_0 \tag{0.5}$$

The total field is then the sum of all the individual contributions:

$$\mathbf{B} = \mathbf{B}_0 \sum_{n=0}^{\infty} \left(\frac{2\mu_0 \chi_m}{3\mu} \right)^n \tag{0.6}$$

$$= \mathbf{B}_0 \frac{1}{1 - \frac{2\mu_0 \chi_m}{3\mu}} \tag{0.7}$$

$$= \mathbf{B}_0 \frac{1}{1 - \frac{2\chi_m}{3(1+\chi_m)}} \tag{0.8}$$

$$= \mathbf{B}_0 \frac{3 + 3\chi_m}{3 + \chi_m} \tag{0.9}$$

Unfortunately, the same method doesn't work for finding the field outside the sphere, since that field is not constant in space.

PINGBACKS

Pingback: Magnetostatic formulas from electrostatic formulas