DIAMAGNETISM

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References: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 6.19.

If a magnetic dipole is placed in an external field, the field exerts a torque on the dipole according to

$$\mathbf{N}=\mathbf{m}\times\mathbf{B} \tag{1}$$

In a magnetic material, this torque tends to line up the electron spins thus producing an overall magnetization in the same direction as the field. This effect is called *paramagnetism*. Because of the Pauli exclusion principle, the electrons in an atom pair up, with the spins of the two electrons within the pair in opposite directions, causing the overall magnetization to cancel out if each atom has an even number of electrons. Paramagnetism is therefore strongest in atoms with odd numbers of electrons.

There is a much weaker effect known as *diamagnetism*, in which the magnetization is opposite to the applied field. This effect occurs in all atoms but in atoms with odd numbers of electrons, it is masked by the stronger paramagnetism, so it is mainly in even-numbered atoms that it appears. Although diamagnetism is ultimately a quantum phenomenon, a rough idea of its cause can be found from a purely classical argument.

Rather than looking at the spin of the electron, we look at its orbital motion. Classically, an electron orbits the nucleus in a circular orbit of radius R at a speed v, giving rise to a steady current $I=-ev/2\pi R$, with $-e$ being the electron's charge. (Technically, a single moving charge doesn't constitute a steady current, but if the electron orbits fast enough, it's a fair approximation.) The dipole moment of a current in a circular loop is

$$\mathbf{m}=\pi R^{2}I\hat{\mathbf{z}}=-\frac{1}{2}evR\hat{\mathbf{z}} \tag{2}$$

where we've taken the plane of the orbit as perpendicular to the z axis.

We want to figure out what changes in the orbit when a field is applied. Without a field, the electron is kept in its orbit by an electrical force producing the centripetal force needed. If the electron is orbiting a nucleus with Z protons, then we get

$$\frac{Ze^2}{4\pi\varepsilon_0 R^2} = m_e \frac{v^2}{R} \tag{3}$$

where m_e is the mass of the electron. (This ignores effects such as the shielding of the nucleus from outer electrons by inner electrons and so on, but since it's a classical argument for a quantum phenomenon, we can't get too picky anyway.)

Now if a field $\mathbf{B}$ in the z direction is applied, then the electron feels an additional force $-e\mathbf{v}\times\mathbf{B}$ which will add to the centripetal force (that is, the magnetic force is towards the centre of the orbit) if the electron is moving so as to give the dipole moment stated above. (If the electron is moving in the opposite direction around its orbit, the magnetic force is outwards.)

This extra centripetal force means that the radius of the orbit and/or the speed of the electron must change. If we assume that the radius stays the same, then the speed must increase, so that, for the new speed $\overline{v}$:

$$\frac{Ze^2}{4\pi\varepsilon_0 R^2} + e\overline{v}B = m_e \frac{\overline{v}^2}{R} \tag{4}$$

Subtracting the first equation from the second, we get

$$e\overline{v}B = \frac{m_e}{R}\left(\overline{v}^2 - v^2\right) = \frac{m_e}{R}\left(\overline{v}+v\right)\left(\overline{v}-v\right) \tag{5}$$

If we now assume that the change in speed is small so that $\overline{v}+v \approx 2\overline{v}$ we get

$$\Delta v = \frac{eBR}{2m_e} > 0 \tag{6}$$

The change in dipole moment is then

$$\Delta\mathbf{m} = -\frac{1}{2}e\Delta vR\hat{\mathbf{z}} = -\frac{e^2R^2B}{4m_e}\hat{\mathbf{z}} \tag{7}$$

That is, the change in dipole moment (which is what causes the magnetization) is opposite to the applied field, which is what is observed in diamagnetism.

If we started with the electron moving in the opposite direction, then the sign of $\mathbf{m}$ is opposite, as is the direction of the magnetic force. If we still assume the radius remains the same, then we must have $\overline{v} < v$ or $\Delta v < 0$. Thus we would then get

$$\Delta\mathbf{m} = \frac{1}{2}e\Delta v R\hat{\mathbf{z}} = -\frac{e^2R^2B}{4m_e}\hat{\mathbf{z}} \tag{8}$$

That is, the change in magnetization is the same as before, so it is still opposite to the applied field.

To get an idea of the numbers involved, we need a value for R. Measured values for elements in the middle of the periodic table are around 1.35×10^{-10} m so using that value, we can calculate the magnetization:

$$\begin{aligned} \mathbf{M} &= \frac{\Delta\mathbf{m}}{\frac{4}{3}\pi R^3} = -\frac{3e^2}{16\pi m_e R}\mathbf{B} && (9) \\ &= -12.46\mathbf{B} && (10) \end{aligned}$$

From this we can get the susceptibility, since $\mathbf{M} = \chi_m\mathbf{H} = \frac{\chi_m}{\mu_0}\mathbf{B}$, so

$$\chi_m = -12.46\mu_0 = -1.57 \times 10^{-5} \tag{11}$$

Typical values as given in Griffiths Table 6.1 are in this general area, which is surprising considering it was a classical derivation.