MAGNETIC TOY

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References: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 6.25.

A magnetic toy consists of some torus-shaped magnets placed on a central pole. (Admittedly, I couldn't find any mention of such a toy, possibly because there isn't a lot you can do with it, so it wouldn't be much fun to play with.) If we treat each torus as an ideal dipole, we can find the distances between successive magnets by equating the magnetic forces on each torus with the gravitational force.

Let's start with just two magnets, each with a dipole moment $\mathbf{m}$ and mass m_d. The first magnet goes at the bottom of the pole and the second magnet is placed on top of it. We've already worked out the force between two dipoles, and we've seen that if the dipole moments are parallel, the force is attractive, so if we want the top magnet to float over the bottom one, we need to align the dipole moments so they are anti-parallel. In this case, the force on the top magnet is

$$\mathbf{F}=\left[\frac{3\mu_0 m^2}{2\pi z^4}-m_d g\right]\hat{\mathbf{z}} \tag{1}$$

Setting this to zero and solving for the height z we get

$$z=\left(\frac{3\mu_0 m^2}{2\pi m_d g}\right)^{1/4} \tag{2}$$

If we now add a third magnet on top of the first two, with its moment parallel to the bottom magnet, then at equilibrium the forces on the middle and top magnets are

$$\mathbf{F}_t=\left[\frac{3\mu_0 m^2}{2\pi}\left(\frac{1}{z_2^4}-\frac{1}{(z_1+z_2)^4}\right)-m_d g\right]\hat{\mathbf{z}}=0 \tag{3}$$

$$\mathbf{F}_m=\left[\frac{3\mu_0 m^2}{2\pi}\left(-\frac{1}{z_2^4}+\frac{1}{z_1^4}\right)-m_d g\right]\hat{\mathbf{z}}=0 \tag{4}$$

where z_2 is the distance between the middle and top magnets and z_1 between the middle and bottom magnets. Since both forces are zero, we can equate them to get

$$\frac{1}{z_2^4} - \frac{1}{(z_1+z_2)^4} = -\frac{1}{z_2^4} + \frac{1}{z_1^4} \tag{5}$$

$$1 - \frac{1}{(\zeta+1)^4} = \frac{1}{\zeta^4} - 1 \tag{6}$$

where $\zeta \equiv z_1/z_2$. We can solve this equation numerically to get

$$\zeta = 0.8501149795\ldots \tag{7}$$

That is, the lower two magnets are slightly closer together than the upper two.