MAGNETIC DIPOLES VERSUS MONOPOLES; AN EXPERIMENT

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References: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 6.28.

An interesting application of the calculation of the magnetic and electric fields within a cavity is the following experiment. We start with a long cylinder of magnetized material with magnetization $\mathbf{M}=M\hat{\mathbf{z}}$. This magnetization produces a bound surface current$\mathbf{K}_b=M\hat{\phi}$ which in turn produces a magnetic field inside the cylinder:

$$\mathbf{B}_m=\mu_0 M\hat{\mathbf{z}} \tag{1}$$

Suppose now that magnetism is caused by magnetic monopoles instead of dipoles produced by microscopic current loops. In that case, the magnetization would induce a bound surface magnetic "charge" density of $\sigma_c=\mathbf{M}\cdot\hat{\mathbf{n}}=M$ on the ends of the cylinder, but no surface charge on the sides. If the ends of the cylinder are small compared to its length, the magnetic field produced by this charge on the ends can be approximated by a point charge, so it will be

$$\mathbf{B}_c=-\frac{\mu_0}{4\pi}\frac{\sigma_c A}{r^2}\hat{\mathbf{z}} \tag{2}$$

where A is the area of the end of the cylinder and r is the distance from the end to the observation point. The minus sign appears because the magnetization causes positive charges to accumulate on the top end of the cylinder so that the magnetic field points downwards, from positive to negative. If $r^2\gg A$, then $\mathbf{B}_c\approx 0$.

Since the fields produced in the two cases are quite different, it should be possible to conduct an experiment to determine which field actually occurs. We can try this by hollowing out a small cavity within the cylinder and inserting a probe to measure the field. We can use our earlier calculations to find the expected field within the cavity in each case.

First, if the field is caused by dipoles, then using $\mathbf{B}_0=\mathbf{B}_m$ for a spherical cavity we get

$$\mathbf{B} = \mathbf{B}_m - \frac{2}{3}\mu_0\mathbf{M} = \frac{1}{3}\mu_0\mathbf{M} \tag{3}$$

If the field is caused by monopoles, then using $\mathbf{B}_0 = \mathbf{B}_c = 0$ we get for a spherical cavity (using the substitutions $\mathbf{E} \to \mathbf{B}$, $\mathbf{P} \to \mathbf{M}$ and $\varepsilon_0 \to 1/\mu_0$):

$$\mathbf{B} = \mathbf{B}_c + \frac{1}{3}\mu_0\mathbf{M} = \frac{1}{3}\mu_0\mathbf{M} \tag{4}$$

Thus for a spherical cavity, the two fields are the same. However, it would seem that if we made the cylinder shorter, then $\mathbf{B}_c$ would be significantly non-zero, so there should be a detectable difference.

For the other two shapes of cavity, we have:

- Needle: $\mathbf{B} = 0$ for both cases.
- Wafer: $\mathbf{B} = \mu_0\mathbf{M}$ for both cases.