OHM'S LAW, CONDUCTIVITY AND RESISTIVITY

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.1.

Up to now, our study of electromagnetism has concentrated on static charge distributions (electrostatics) or steady current distributions (giving rise to magnetostatics). Now we start looking in more detail at what happens when currents flow in arbitrary ways.

To begin, we can state the relation between electromagnetic force and current density:

$$\mathbf{J}=\sigma\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right) \tag{1}$$

where σ is the *conductivity* of the material in which the current is flowing. This is an empirical relation, based on observation, rather than a theoretically derived result. Note that σ here is *not* surface charge density; it is the standard notation for conductivity. Its inverse is $\rho=1/\sigma$, called the *resistivity*, and again should not be confused with volume charge density. Typically, the velocity $\mathbf{v}$ of the charges is so small that the magnetic force term can be neglected.

As an example, suppose we have two concentric conducting spheres of radii a and b with $b>a$, held at a potential difference of V. The area between the spheres is filled with a conducting material with conductivity σ. Our problem is to determine the current that flows between the spheres.

The electric field between the spheres is due entirely to the inner sphere, which we'll assume has a surface charge density of s. Then the field between the spheres is

$$\mathbf{E}=\frac{4\pi a^2 s}{4\pi\varepsilon_0}\frac{\hat{\mathbf{r}}}{r^2}=\frac{a^2 s}{\varepsilon_0}\frac{\hat{\mathbf{r}}}{r^2} \tag{2}$$

Ignoring the magnetic term, the current density is

$$\mathbf{J}=\sigma\mathbf{E}=\frac{a^2 s\sigma}{\varepsilon_0}\frac{\hat{\mathbf{r}}}{r^2} \tag{3}$$

and the total current between the spheres is

$$I=\int\mathbf{J}\cdot d\mathbf{a}=\frac{a^{2}s\sigma}{\varepsilon_{0}}\int_{0}^{2\pi}\int_{0}^{\pi}\frac{r^{2}\sin\theta}{r^{2}}d\theta d\phi=\frac{4\pi a^{2}s\sigma}{\varepsilon_{0}}\tag{4}$$

where the integral is done over a sphere of radius r such that $a<r<b$.

We still need to get rid of the explicit reference to the surface charge density s, which we can do by imposing the condition that the potential difference between the spheres is V. We have

$$V=-\int_{a}^{b}\mathbf{E}\cdot d\ell=-\frac{sa^{2}}{\varepsilon_{0}}\int_{a}^{b}\frac{dr}{r^{2}}=\frac{sa^{2}}{\varepsilon_{0}}\left(\frac{1}{a}-\frac{1}{b}\right)=\frac{sa\left(b-a\right)}{\varepsilon_{0}b}\tag{5}$$

Thus the surface charge density is

$$s=\frac{\varepsilon_{0}bV}{a\left(b-a\right)}\tag{6}$$

and the current is

$$I=\frac{4\pi ab\sigma}{b-a}V\tag{7}$$

Note that the current is proportional to the potential difference, which is often true in resistive materials and is known as *Ohm's law,* which is more usually written as

$$V=IR\tag{8}$$

where R is the resistance of the material. In this case

$$R=\frac{b-a}{4\pi ab\sigma}\tag{9}$$

For large b, $I\to4\pi a\sigma V$ and $R\to1/4\pi a\sigma$. This is presumably because there is much more conducting material at a large radius, so it contributes less to the total resistance between the spheres.

A setup used to measure the conductivity of sea water is to place two conducting spheres each of radius a a large distance apart (say, at opposite ends of a ship) in the water and pass current between them by holding one sphere at potential zero and the other at potential V. (I'm not 100% convinced of this argument, but here goes arrangement as two instances of the concentric sphere setup in this problem. The sphere at potential V can be viewed as concentric spheres with the one at a at potential V and a distant outer sphere at potential $V/2$. The other sphere (at potential 0) can be viewed as an inner sphere at potential 0 and an outer distant one at potential $V/2$. The currents within both pair of spheres must be equal (since there is a constant overall current passing from V to 0), so if we look at the first sphere, the current is (taking $b\to\infty$):

$$I=4\pi a\sigma\frac{V}{2}=2\pi a\sigma V \tag{10}$$

which must also be the current passing into the second sphere. Thus the conductivity of the sea water can be measured as

$$\sigma=\frac{I}{2\pi aV} \tag{11}$$

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