CHARGING AND DISCHARGING A CAPACITOR: APPLYING OHM'S LAW

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.2.

As a simple example of Ohm's law, suppose we have a capacitor C with an initial charge of Q_0 connected in a circuit with a resistor R. The voltage across the capacitor is $V(t) = Q(t)/C$ which must balance the voltage across the resistor, so that

$$\frac{Q(t)}{C} = -I(t)R = -\dot{Q}R \tag{0.1}$$

The differential equation has the solution

$$\begin{aligned} Q(t) &= Q_0 e^{-t/RC} && (0.2)\\ &= CV_0 e^{-t/RC} && (0.3) \end{aligned}$$

where V_0 is the initial voltage across the capacitor. The current is then

$$I(t) = -\frac{V_0}{R} e^{-t/RC} \tag{0.4}$$

where the minus sign indicates that the direction of the current is such as to discharge the capacitor.

The initial energy stored in the capacitor is $W_C = \frac{1}{2}CV_0^2$ and the total energy dissipated as heat in the resistor is

$$\begin{aligned} W_R &= \int_0^\infty I^2(t)R\,dt && (0.5)\\ &= \frac{V_0^2}{R}\int_0^\infty e^{-2t/RC}dt && (0.6)\\ &= \frac{1}{2}CV_0^2 && (0.7) \end{aligned}$$

Thus the two energies balance out.

Suppose we now start with an uncharged capacitor and connect it and the resistor in series with a battery that is held at constant potential V_0. In this case, the sum of the voltages across C and R must add up to V_0, so we have

$$V_0 = \frac{Q(t)}{C} + \dot{Q}R \tag{0.8}$$

The solution of this differential equation with initial condition $Q(0) = 0$ is

$$Q(t) = CV_0\left(1 - e^{-t/RC}\right) \tag{0.9}$$

$$I(t) = \dot{Q}(t) = \frac{V_0}{R}e^{-t/RC} \tag{0.10}$$

This time, the total energy delivered by the battery is the sum of the energy stored in the capacitor and the energy dissipated as heat in the resistor. Because the voltage is constant, the power generated is $I(t)V_0$ and the energy from the battery is

$$W_B = \int_0^\infty I(t)V_0 dt \tag{0.11}$$

$$= \frac{V_0^2}{R}\int_0^\infty e^{-t/RC}dt \tag{0.12}$$

$$= CV_0^2 \tag{0.13}$$

Thus the battery delivers twice the energy that ends up stored in the capacitor. That it is independent of R isn't terribly surprising; with a larger resistance it just takes longer to charge the capacitor, but the total energy used to do it is the same.

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