

CHARGING AND DISCHARGING A CAPACITOR: GENERAL CASE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.3.

The problem of charging and discharging a capacitor can be generalized to two chunks of conducting material of arbitrary shape embedded within a medium with conductivity σ . First, we can work out the resistance between the two conductors. Suppose at some instant in time the potential difference between the two conductors is V , and the current flowing between them is I . This current is the surface integral of the current density over some area that encloses one of the conductors; let's make it the conductor with a net positive charge Q on its surface. That is

$$(0.1) \quad I = \int \mathbf{J} \cdot d\mathbf{a}$$

By our definition of conductivity, we're taking $\mathbf{J} = \sigma\mathbf{E}$ so

$$(0.2) \quad I = \sigma \int \mathbf{E} \cdot d\mathbf{a} = \frac{\sigma Q}{\epsilon_0}$$

using Gauss's law. The capacitance of the system is given by $C = Q/V$ and Ohm's law says that $V = IR$ so

$$(0.3) \quad I = \frac{V}{R} = \frac{Q}{CR} = \frac{\sigma Q}{\epsilon_0}$$

$$(0.4) \quad R = \frac{\epsilon_0}{\sigma C}$$

Given the resistance between the conductors, the problem of finding the charge, current and potential as functions of time reduces to that in the previous post. So we have

$$(0.5) \quad Q(t) = CV_0 e^{-t/RC}$$

$$(0.6) \quad = CV_0 e^{-\sigma t/\epsilon_0}$$

$$(0.7) \quad I(t) = \dot{Q}(t) = -\frac{CV_0\sigma}{\epsilon_0} e^{-\sigma t/\epsilon_0}$$

$$(0.8) \quad V(t) = -I(t)R = \frac{RCV_0\sigma}{\epsilon_0} e^{-\sigma t/\epsilon_0} = V_0 e^{-\sigma t/\epsilon_0}$$

Charging the capacitor with a battery of fixed potential V_0 gives the same results as in the previous post with $RC = \epsilon_0/\sigma$:

$$(0.9) \quad Q(t) = CV_0 \left(1 - e^{-\sigma t/\epsilon_0}\right)$$

$$(0.10) \quad I(t) = \dot{Q}(t) = \frac{V_0}{R} e^{-\sigma t/\epsilon_0} = \frac{\sigma CV_0}{\epsilon_0} e^{-\sigma t/\epsilon_0}$$

$$(0.11) \quad V(t) = \frac{Q(t)}{C} = V_0 \left(1 - e^{-\sigma t/\epsilon_0}\right)$$

The voltage across the resistor is $V_R(t) = V_0 - V(t) = V_0 e^{-\sigma t/\epsilon_0} = I(t)R$.

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