CHARGING AND DISCHARGING A CAPACITOR: GENERAL CASE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.3.

The problem of charging and discharging a capacitor can be generalized to two chunks of conducting material of arbitrary shape embedded within a medium with conductivity σ. First, we can work out the resistance between the two conductors. Suppose at some instant in time the potential difference between the two conductors is V, and the current flowing between them is I. This current is the surface integral of the current density over some area that encloses one of the conductors; let's make it the conductor with a net positive charge Q on its surface. That is

$$I=\int\mathbf{J}\cdot d\mathbf{a} \tag{1}$$

By our definition of conductivity, we're taking $\mathbf{J}=\sigma\mathbf{E}$ so

$$I=\sigma\int\mathbf{E}\cdot d\mathbf{a}=\frac{\sigma Q}{\epsilon_0} \tag{2}$$

using Gauss's law. The capacitance of the system is given by $C=Q/V$ and Ohm's law says that $V=IR$ so

$$\begin{aligned} I &= \frac{V}{R}=\frac{Q}{CR}=\frac{\sigma Q}{\epsilon_0} && (3)\\ R &= \frac{\epsilon_0}{\sigma C} && (4) \end{aligned}$$

Given the resistance between the conductors, the problem of finding the charge, current and potential as functions of time reduces to that in the previous post. So we have

$$\begin{aligned} Q(t) &= CV_0 e^{-t/RC} & (5)\\ &= CV_0 e^{-\sigma t/\epsilon_0} & (6)\\ I(t) &= \dot{Q}(t) = -\frac{CV_0\sigma}{\epsilon_0} e^{-\sigma t/\epsilon_0} & (7)\\ V(t) &= -I(t)R = \frac{RCV_0\sigma}{\epsilon_0} e^{-\sigma t/\epsilon_0} = V_0 e^{-\sigma t/\epsilon_0} & (8) \end{aligned}$$

Charging the capacitor with a battery of fixed potential V_0 gives the same results as in the previous post with $RC = \epsilon_0/\sigma$:

$$\begin{aligned} Q(t) &= CV_0\left(1 - e^{-\sigma t/\epsilon_0}\right) & (9)\\ I(t) &= \dot{Q}(t) = \frac{V_0}{R} e^{-\sigma t/\epsilon_0} = \frac{\sigma C V_0}{\epsilon_0} e^{-\sigma t/\epsilon_0} & (10)\\ V(t) &= \frac{Q(t)}{C} = V_0\left(1 - e^{-\sigma t/\epsilon_0}\right) & (11) \end{aligned}$$

The voltage across the resistor is $V_R(t) = V_0 - V(t) = V_0 e^{-\sigma t/\epsilon_0} = I(t)R$.

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