OHM'S LAW WITH VARIABLE CONDUCTIVITY

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.4.

Here's an example of using Ohm's law to calculate the resistance of a system with variable conductivity. We have a coaxial cylinder of length L with an inner cylinder of radius a and an outer cylinder of radius b. The conductivity is $\sigma(r) = k/r$ where k is a constant. If the two cylinders are held at a constant potential difference, then the current passing between them is also constant I. We can get I by integrating over a cylindrical surface between the two cylinders:

$$I = \sigma(r) \int \mathbf{E} \cdot d\mathbf{a} \tag{1}$$

By symmetry (ignoring end effects, or else considering a length L within a much longer cylinder), $\mathbf{E}$ is radial and can depend at most on r, so if we choose an integration cylinder of a fixed radius r then

$$\begin{aligned} I &= \frac{k}{r} 2\pi r E L = 2\pi k L E && (2) \\ E &= \frac{I}{2\pi k L} && (3) \end{aligned}$$

The field is therefore constant between the cylinders, which means the potential difference between the cylinders is just

$$V = E(b-a) = \frac{I}{2\pi k L}(b-a) \tag{4}$$

giving a resistance of (using Ohm's law $V = IR$):

$$R = \frac{b-a}{2\pi k L} \tag{5}$$