

## OHM'S LAW WITH VARIABLE CONDUCTIVITY

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.4.

Here's an example of using Ohm's law to calculate the resistance of a system with variable conductivity. We have a coaxial cylinder of length  $L$  with an inner cylinder of radius  $a$  and an outer cylinder of radius  $b$ . The conductivity is  $\sigma(r) = k/r$  where  $k$  is a constant. If the two cylinders are held at a constant potential difference, then the current passing between them is also constant  $I$ . We can get  $I$  by integrating over a cylindrical surface between the two cylinders:

$$I = \sigma(r) \int \mathbf{E} \cdot d\mathbf{a} \quad (1)$$

By symmetry (ignoring end effects, or else considering a length  $L$  within a much longer cylinder),  $\mathbf{E}$  is radial and can depend at most on  $r$ , so if we choose an integration cylinder of a fixed radius  $r$  then

$$I = \frac{k}{r} 2\pi r E L = 2\pi k L E \quad (2)$$

$$E = \frac{I}{2\pi k L} \quad (3)$$

The field is therefore constant between the cylinders, which means the potential difference between the cylinders is just

$$V = E(b-a) = \frac{I}{2\pi k L} (b-a) \quad (4)$$

giving a resistance of (using Ohm's law  $V = IR$ ):

$$R = \frac{b-a}{2\pi k L} \quad (5)$$