OHM'S LAW WITH VARIABLE CONDUCTIVITY

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.4.

Here's an example of using Ohm's law to calculate the resistance of a system with variable conductivity. We have a coaxial cylinder of length L with an inner cylinder of radius a and an outer cylinder of radius b. The conductivity is $\sigma(r) = k/r$ where k is a constant. If the two cylinders are held at a constant potential difference, then the current passing between them is also constant I. We can get I by integrating over a cylindrical surface between the two cylinders:

$$I = \sigma(r)\int \mathbf{E}\cdot d\mathbf{a} \tag{0.1}$$

By symmetry (ignoring end effects, or else considering a length L within a much longer cylinder), $\mathbf{E}$ is radial and can depend at most on r, so if we choose an integration cylinder of a fixed radius r then

$$I = \frac{k}{r}2\pi rEL = 2\pi kLE \tag{0.2}$$

$$E = \frac{I}{2\pi kL} \tag{0.3}$$

The field is therefore constant between the cylinders, which means the potential difference between the cylinders is just

$$V = E(b-a) = \frac{I}{2\pi kL}(b-a) \tag{0.4}$$

giving a resistance of (using Ohm's law $V = IR$):

$$R = \frac{b-a}{2\pi kL} \tag{0.5}$$