PERPETUAL MOTION IN AN ELECTRIC CIRCUIT?

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.6.

Suppose we try to build a perpetual motion machine as follows. We take a parallel plate capacitor and charge it up so that the field between the plates is $\mathbf{E}$. We'll take the plates to lie parallel to the xy plane, so that $\mathbf{E}$ points in the z direction. Now we take a rectangular loop of wire and align it so that it lies in the xz plane with one side (of length h) of the rectangle between the plates, and the other end outside the plates. Thus between the plates, $\mathbf{E}$ points along one edge of the rectangle and is perpendicular to the two edges that lead into and out of the capacitor.

A naive calculation now tells us that the electromotive force in the loop should be

$$\mathcal{E}=\oint\mathbf{E}\cdot d\ell=Eh$$

since the field $\mathbf{E}$ is the force per unit charge, and $\mathbf{E}$ is parallel to the side of the rectangle inside the plates, is perpendicular to the two sides leading into and out of the plates, and the field is zero on the fourth edge since it lies outside the plates. If the wire has a resistance R, this emf would therefore give rise to a current $I=\mathcal{E}/R=Eh/R$. Since the capacitor never discharges, this current appears to be conjured out of nothing and would flow forever.

Clearly this is impossible, since it would result in a steady dissipation of energy as heat from the resistor, which violates conservation of energy. The resolution of the paradox lies in the fact that the field between the plates is not exactly perpendicular to the plates, especially near the edges, and there is also a fringe field that extends outside the plates. To see that this gives an emf that is in the right direction to counter that generated by the side of length h inside the plates, consider a point just outside the plates on the x axis. If the bottom plate is positive and the top negative, then the field at this point bends outwards from the lower plate and curves round to bend back inwards towards the top plate. If the path of integration is clockwise looking down on the xz plane, then $\mathbf{E}\cdot d\ell$ is negative on both the lower and upper edges (since the angle between $\mathbf{E}$ and $d\ell$ lies between $\frac{\pi}{2}$ and π in both cases), while $\mathbf{E}\cdot d\ell$ is positive on the vertical side of length

h. Obviously we'd need to do a complete analysis to actually prove that these two contributions to the emf cancel out (and that's not easy), but we know they must since the integral $\oint \mathbf{E}\cdot d\ell = 0$ for all closed loops in an electrostatic system (a consequence of Stokes's law and the fact that $\nabla\times\mathbf{E} = 0$ in electrostatics).

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