

## EMF NEAR AN INFINITE WIRE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.8.

Here's a simple illustration of calculating motional emf from the change in magnetic flux. An infinite straight wire lies along the  $x$  axis and carries a current  $I$  in the  $+x$  direction. A square loop of wire of side length  $a$  lies in the first quadrant of the  $xy$  plane with its sides parallel to the  $x$  and  $y$  axes, and with the nearest edge a distance  $s$  from the infinite wire.

The magnetic field from the infinite wire is, at a distance  $r$  from the wire:

$$(0.1) \quad \mathbf{B} = \frac{I\mu_0}{2\pi r} \hat{\phi}$$

that is, it circles the wire in a clockwise direction looking towards the  $+x$  direction. The magnetic flux is then

$$(0.2) \quad \Phi = \frac{\mu_0 I}{2\pi} \int_s^{s+a} \frac{adr}{r}$$

$$(0.3) \quad = \frac{\mu_0 I a}{2\pi} \ln \frac{s+a}{s}$$

If the loop is now moved away from the wire at a constant speed  $v$ , then the emf is

$$(0.4) \quad \mathcal{E} = -\frac{d\Phi}{dt}$$

$$(0.5) \quad = -\frac{\mu_0 I a}{2\pi} \frac{s}{s+a} \left( -\frac{a}{s^2} \right) \frac{ds}{dt}$$

$$(0.6) \quad = \frac{\mu_0 I a^2 v}{2\pi s(s+a)}$$

To work out the direction of the current, we note that from the right hand rule, the magnetic force is in the  $+x$  direction on both the edge at distance  $s$  and the edge at distance  $s+a$  but since the closer edge experiences the stronger field, the net current is in the  $+x$  direction on the closer edge so the current flows counterclockwise as seen from above the  $xy$  plane.

If the loop is moved parallel to the wire (that is, keeping  $s$  constant), there is a force along the two edges perpendicular to the wire, but because the magnetic field at a given distance from the wire is always the same, there is no net current and the generated emf is zero.