EMF NEAR AN INFINITE WIRE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.8.

Here's a simple illustration of calculating motional emf from the change in magnetic flux. An infinite straight wire lies along the x axis and carries a current I in the $+x$ direction. A square loop of wire of side length a lies in the first quadrant of the xy plane with its sides parallel to the x and y axes, and with the nearest edge a distance s from the infinite wire.

The magnetic field from the infinite wire is, at a distance r from the wire:

$$\mathbf{B}=\frac{I\mu_0}{2\pi r}\hat{\phi} \tag{1}$$

that is, it circles the wire in a clockwise direction looking towards the $+x$ direction. The magnetic flux is then

$$\begin{aligned}\Phi &= \frac{\mu_0 I}{2\pi}\int_s^{s+a}\frac{a\,dr}{r} && (2)\\ &= \frac{\mu_0 I a}{2\pi}\ln\frac{s+a}{s} && (3)\end{aligned}$$

If the loop is now moved away from the wire at a constant speed v, then the emf is

$$\begin{aligned}\mathcal{E} &= -\frac{d\Phi}{dt} && (4)\\ &= -\frac{\mu_0 I a}{2\pi}\frac{s}{s+a}\left(-\frac{a}{s^2}\right)\frac{ds}{dt} && (5)\\ &= \frac{\mu_0 I a^2 v}{2\pi s(s+a)} && (6)\end{aligned}$$

To work out the direction of the current, we note that from the right hand rule, the magnetic force is in the $+x$ direction on both the edge at distance s and the edge at distance $s+a$ but since the closer edge experiences the stronger field, the net current is in the $+x$ direction on the closer edge so the current flows counterclockwise as seen from above the xy plane.

If the loop is moved parallel to the wire (that is, keeping s constant), there is a force along the two edges perpendicular to the wire, but because the magnetic field at a given distance from the wire is always the same, there is no net current and the generated emf is zero.