BALANCING MAGNETIC FORCE WITH GRAVITY

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.11.

Here's an illustration of how a magnetic force can balance gravity, even for a non-magnetic substance. We have a square loop of aluminum (or aluminium, if you must) with a constant cross-sectional area α and side length a. It is dropped from rest between the poles of a strong magnet with a field strength of $B=1$ Tesla, so that the field is horizontal, and the top edge of the square is within the field, while the bottom edge is outside the field. As we've seen in a similar problem, once the square has a non-zero downwards velocity v, a magnetic force arises that opposes the fall (so it acts upwards), and that force has magnitude

$$F_{mag}=\frac{a^2B^2v}{R} \tag{1}$$

where R is the resistance of the aluminum loop. The net force on the loop is then

$$m\frac{dv}{dt}=mg-\frac{a^2B^2v}{R} \tag{2}$$

where m is the mass of the loop and g is the acceleration of gravity.

The system reaches terminal velocity when $dv/dt=0$, so

$$v_{term}=\frac{mgR}{a^2B^2} \tag{3}$$

To reduce this to physically measurable quantities we first note that the resistance of a wire of uniform cross section is given by Griffiths in his example 7.1 and is

$$R=\frac{\rho L}{\alpha}=\frac{4a\rho}{\alpha} \tag{4}$$

where ρ is the resistivity of the wire and L is its length. The mass of the wire is

$$m = \alpha L d = 4a\alpha d \tag{5}$$

where d is the mass density of the wire. Therefore

$$v_{term} = \frac{16\rho g d}{B^2} \tag{6}$$

For aluminum, the values are

$$\begin{aligned} \rho &= 2.82 \times 10^{-8} \text{ ohm m} && (7)\\ d &= 2700 \text{ kg m}^{-3} && (8)\\ g &= 9.8 \text{ m s}^{-2} && (9) \end{aligned}$$

Plugging in the numbers gives

$$v_{term} = 0.0119 \text{ m s}^{-1} \tag{10}$$

By the way, to check that the units are correct it's worth noting that

$$\begin{aligned} 1 \text{ ohm} &= 1 \text{ kg m}^2 \text{ s}^{-3} \text{ Amp}^{-2} && (11)\\ 1 \text{ Tesla} &= 1 \text{ kg s}^{-2}\text{Amp}^{-1} && (12) \end{aligned}$$

Plugging these into the formula for v_{term} does indeed give units of velocity.

We can work out the time required to achieve 90% of terminal velocity by solving the differential equation 2. The solution turns out to be

$$\begin{aligned} v(t) &= \frac{mgR}{a^2B^2}\left(1 - e^{-ta^2B^2/Rm}\right) && (13)\\ &= v_{term}\left(1 - e^{-gt/v_{term}}\right) && (14) \end{aligned}$$

The 90% time is reached when

$$\begin{aligned} \frac{v(t_{90})}{v_{term}} &= 0.9 && (15)\\ e^{-gt_{90}/v_{term}} &= 0.1 && (16)\\ t_{90} &= -\frac{v_{term}}{g}\ln 0.1 && (17)\\ &= 2.8 \times 10^{-3} \text{ s} && (18) \end{aligned}$$

Thus when a square loop of a conductor (since most conductors have similarly small resistivities) is dropped in a strong magnetic field, it will drift down at a constant speed almost immediately.

If the loop were cut, no current could flow, so there would be no magnetic force to oppose gravity, and the loop would fall at the normal rate.

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