LENZ'S LAW

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.13.

Faraday's law states that the emf generated in a circuit is determined by the rate of change of magnetic flux through that circuit, according to

$$\mathscr{E}=\oint\mathbf{E}\cdot d\ell=-\frac{d\Phi}{dt} \tag{0.1}$$

Since $\Phi=\int\mathbf{B}\cdot d\mathbf{a}$ then if the area enclosed by the loop stays the same, we have

$$\oint\mathbf{E}\cdot d\ell=-\frac{d\Phi}{dt}=-\int\frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{a} \tag{0.2}$$

Although the direction of the induced emf can be determined by the Lorentz force law in cases where the loop moves relative to the magnetic field, this doesn't work if both loop and magnet are at rest and only the field strength changes. A simple criterion known as *Lenz's law* can be used in all cases to determine the direction of the emf. Lenz's law states that the induced emf always tends to oppose the flux change that produces it.

As a simple example, suppose we have a square loop of side length a in the first quadrant of the xy plane, with one corner at the origin and two sides lying along the x and y axes. A magnetic field $\mathbf{B}=ky^3t^2\hat{\mathbf{z}}$ (where k is a positive constant) is applied to the loop. The flux through the loop is then

$$\Phi=kt^2a\int_0^a y^3dy=\frac{1}{4}kt^2a^5 \tag{0.3}$$

and the emf is

$$\mathscr{E}=-\frac{d\Phi}{dt}=-\frac{1}{2}kta^5 \tag{0.4}$$

Since the magnetic field monotonically increases with time, the flux is also increasing and is positive (where the positive direction is taken to be the $+z$ direction). By Lenz's law, the emf must oppose this flux so the current must generate a field that points downward, which means the current flows clockwise around the loop as seen from above.

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