

COAXIAL CABLE WITH VARYING CURRENT

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.16.

Here's another (mis)application of Faraday's law, using the quasistatic approximation to calculate the fields involved. This time we have a coaxial cable in which the current flows in the $+z$ direction on the inner wire and back in the $-z$ direction on the coaxial cylinder. The current varies with time according to

$$(0.1) \quad I(t) = I_0 \cos \omega t$$

The magnetic field due to the inner wire is given by

$$(0.2) \quad \mathbf{B}_w(r, t) = \hat{\phi} \frac{\mu_0 I_0}{2\pi r} \cos \omega t$$

The magnetic field due to the coaxial cylinder is just the negative of this outside the cylinder, and zero inside:

$$(0.3) \quad \mathbf{B}_c(r, t) = \begin{cases} -\hat{\phi} \frac{\mu_0 I_0}{2\pi r} \cos \omega t & r > a \\ 0 & r < a \end{cases}$$

Therefore the net magnetic field outside the cylinder is always zero, and the field inside is given by \mathbf{B}_w alone. The induced electric field is parallel to the z axis, as can be seen by a similar argument to that given for the magnetic field of a solenoid. First, \mathbf{E} cannot have a radial component, since if we reversed the directions of the currents, then \mathbf{B} and hence \mathbf{E} reverse their directions. But reversing the currents is equivalent to turning the wire through 180 degrees, and that shouldn't affect the radial component of the field, so $E_r = 0$.

\mathbf{E} also cannot have a ϕ component. Since \mathbf{B} is entirely in the ϕ direction, so is $-\partial\mathbf{B}/\partial t$, so if we choose a path of integration that is a circle at constant r , and a corresponding surface of integration that is perpendicular to the z direction (that is, to the cable), then $d\mathbf{a}$ is always perpendicular to $-\partial\mathbf{B}/\partial t$ so the integral on the right is zero:

$$(0.4) \quad \oint \mathbf{E} \cdot d\ell = - \int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a}$$

By symmetry, E_ϕ must be constant around the circle so we get $\oint \mathbf{E} \cdot d\ell = 2\pi r E_\phi = 0$, so $E_\phi = 0$. Therefore, \mathbf{E} must have a z component only.

To figure out the actual value of \mathbf{E} , we can use the same argument as in calculating the solenoid's magnetic field. We choose a square loop with side length ℓ that lies in the yz plane. First, we take the loop to lie entirely outside the coaxial cable, where $\mathbf{B} = 0$ always. This means that (taking the near edge to be a distance s from the cable's axis):

$$(0.5) \quad \oint \mathbf{E} \cdot d\ell = \ell (E(s+\ell) - E(s)) = 0$$

If we assume that $\mathbf{E} \rightarrow 0$ at infinity, then $\mathbf{E} = 0$ everywhere outside the cable.

To find \mathbf{E} inside the cable, we can take the loop of integration to have its inner edge at distance s from the axis, where $0 < s < a$, and the outer edge outside the cylinder, where $\mathbf{E} = 0$. Then

$$(0.6) \quad \oint \mathbf{E} \cdot d\ell = \ell E(s)$$

$$(0.7) \quad = - \int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a}$$

$$(0.8) \quad = \frac{\ell \omega \mu_0 I_0}{2\pi} \sin \omega t \int_s^a \frac{dr}{r}$$

$$(0.9) \quad = \frac{\ell \omega \mu_0 I_0}{2\pi} \sin \omega t \ln \frac{a}{s}$$

To get the direction of \mathbf{E} we use Lenz's law and the right hand rule. If the current flows in the $+z$ direction and is increasing, then the magnetic field is in the $+\phi$ direction and increasing, so the flux through the loop is increasing. The induced electric field will oppose this increase, which means that \mathbf{E} inside the cylinder must point in the $-z$ direction. Conversely, if I is decreasing, \mathbf{E} points in the $+z$ direction. Since I starts off at its maximum value at $t = 0$, the current is decreasing initially, so the initial value of \mathbf{E} must point in the $+z$ direction, thus:

$$(0.10) \quad \mathbf{E}(s,t) = \begin{cases} \hat{\mathbf{z}} \frac{\omega \mu_0 I_0}{2\pi} \sin \omega t \ln \frac{a}{s} & 0 < s < a \\ 0 & s > a \end{cases}$$

As always, we need to issue a caution that this analysis is deeply flawed, since we're applying the quasistatic approximation to a cable of infinite length, and we're assuming that the current can change everywhere along the length of the cable at the same rate, which doesn't happen in real life.

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