FARADAY'S LAW: WIRE LOOP ENCIRCLING A SOLENOID

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.17.

Here's another example of Faraday's law, using the quasistatic approximation involving a solenoid. A solenoid of radius a with its axis along the z axis has n turns per unit length carrying current in the $+\phi$ direction. The current I is increasing at a constant rate, such that $\dot{I}=k$. A loop of wire with resistance R encircles the solenoid and we want to find the induced current I_R in the wire.

By Faraday's law, the emf around the loop depends only on change in flux enclosed by the loop (note that it doesn't depend on the shape of the loop). Thus we have

$$\mathscr{E} = -\frac{d\Phi}{dt}=-\pi a^2\mu_0 nk \tag{1}$$

$$I_R = \frac{\mathscr{E}}{R}=-\frac{\pi a^2\mu_0 nk}{R} \tag{2}$$

Since Lenz's law tells us the induced current opposes the change in flux, the current in the wire flows in the $-\phi$ direction.

Now suppose the current in the solenoid is held constant, and the solenoid is removed from the loop and reinserted in the opposite direction. How much charge flows through the resistor in total during this process? Charge is the integral of current over time, so it might appear that we would need to know the rate at which the solenoid is moved out and replaced, but in fact we don't. Consider the stage where the solenoid is removed from the loop. The charge through the resistor due to this stage is

$$Q=\int I_R(t)\,dt=-\frac{1}{R}\int\frac{d\Phi}{dt}=-\frac{1}{R}\Delta\Phi \tag{3}$$

where $\Delta\Phi$ is the change in flux before and after the solenoid is removed. That is, the details of how fast the solenoid is moved don't matter; all that matters is the start and end values of the flux. But the start value of the flux is $\Phi_s=\pi a^2\mu_0 nI$ and the end value is $\Phi_e=0$, so the magnitude of charge moved is

$$Q=\frac{\pi a^{2}\mu_{0}nI}{R} \tag{4}$$

The process of reinserting the solenoid the other way round will move the same amount of charge, so the total charge through the resistor is

$$Q_{T}=\frac{2\pi a^{2}\mu_{0}nI}{R} \tag{5}$$

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