FARADAY'S LAW: CUTTING A CURRENT-CARRYING WIRE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.18.

And yet another example of Faraday's law, using the quasistatic approximation in a situation similar to the last problem. This time we have an infinite wire carrying a current I in the $+z$ direction and a square wire loop of side length a in the xz plane, with its nearest edge parallel to and at a distance s from the wire. The wire is suddenly cut so that the current drops to zero. What can we say about the current and charge flowing through the loop?

The magnetic field produced by the wire is

$$\mathbf{B}(r,t)=\hat{\phi}\frac{\mu_0 I(t)}{2\pi r} \tag{1}$$

so the field points in the $+y$ direction through the loop. If we take the area element to also point in the $+y$ direction, then the flux is

$$\Phi(t)=\int\mathbf{B}\cdot d\mathbf{a}=\frac{\mu_0 aI(t)}{2\pi}\int_s^{s+a}\frac{dr}{r}=\frac{\mu_0 aI(t)}{2\pi}\ln\frac{s+a}{s} \tag{2}$$

where $I(t)$ is the current as a function of time; the current will drop sharply to zero when the wire is cut, but of course it won't be a step function in real life.

When the wire is cut, the flux will suddenly decrease to zero, so the induced current will oppose this, and the current flows in a counterclockwise direction (that is, the current in the side of the square closest to the wire will be in the $+z$ direction). The actual magnitude of the current depends on the resistance R in the wire and how fast the current drops to zero, so we can't specify those with the information given.

The total charge that moves through the loop is, as in the previous example, determined from the flux difference before and after the cut.

$$|Q|=\frac{1}{R}\Delta\Phi=\frac{\mu_0 aI}{2\pi R}\ln\frac{s+a}{s} \tag{3}$$