

FARADAY'S LAW AND THE BIOT-SAVART LAW

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.19.

As an example of the analogy between Faraday's law and Ampère's law, suppose we have a toroidal solenoid with inner radius a , width w and height h , where both w and h are much less than a . The solenoid has N turns in total, and carries a current I that is increasing at the constant rate of $\dot{I} = k$. The magnetic field inside the solenoid is given by Griffiths in his example 5.10 and is

$$(1) \quad B = \frac{\mu_0 N I}{2\pi r}$$

where r is the radial distance. In our case we can approximate r by the constant a since the torus is very thin compared to its radius, so we get

$$(2) \quad B \approx \frac{\mu_0 N I}{2\pi a}$$

$$(3) \quad \Phi \approx \frac{\mu_0 N I h w}{2\pi a}$$

With this approximation, the change in flux is

$$(4) \quad \frac{d\Phi}{dt} = \frac{\mu_0 N k h w}{2\pi a}$$

Since the magnetostatic case and the Faraday case are mathematically equivalent (provided there is no free charge), we can use the Biot-Savart law to calculate the electric field generated by a steady change in magnetic flux if we replace \mathbf{J} by $-\frac{1}{\mu_0} \frac{\partial \mathbf{B}}{\partial t}$, or, in the case of a linear current in the magnetostatic case, we replace I by $-\frac{1}{\mu_0} \frac{\partial \Phi}{\partial t}$.

For example, we can find the electric field at a point on the axis of the torus by using the Biot-Savart law:

$$(5) \quad \mathbf{E} = -\frac{1}{4\pi} \frac{\partial \Phi}{\partial t} \int_1 \frac{d\ell' \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}$$

Griffiths works this out in his example 5.6, and the derivation goes like this: the line segment $d\ell'$ is always in the xy plane and is tangent to the torus. The vector $\mathbf{r} - \mathbf{r}'$ points from a point on the torus to a point on the z axis, so it is always perpendicular to $d\ell'$. The cross product $d\ell' \times (\mathbf{r} - \mathbf{r}')$ makes a constant angle θ with the z axis, and θ is also the angle between $\mathbf{r} - \mathbf{r}'$ and the xy plane. By symmetry, any x and y components of the cross product will cancel out in the integration, so we're left with only the z component, which is $|\mathbf{r} - \mathbf{r}'| \cos \theta d\ell$. Furthermore, the magnitude of $\mathbf{r} - \mathbf{r}'$ is always the same, and is $|\mathbf{r} - \mathbf{r}'| = \sqrt{a^2 + z^2}$, and $\cos \theta = a/\sqrt{a^2 + z^2}$, so

$$(6) \quad \int \frac{d\ell' \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} = 2\pi a \cos \theta \frac{\sqrt{a^2 + z^2}}{(a^2 + z^2)^{3/2}} \hat{\mathbf{z}} = \frac{2\pi a^2}{(a^2 + z^2)^{3/2}} \hat{\mathbf{z}}$$

and the electric field is then

$$(7) \quad \mathbf{E} = -\frac{\partial \Phi}{\partial t} \frac{a^2}{2(a^2 + z^2)^{3/2}} \hat{\mathbf{z}} = -\frac{\mu_0 N k h w}{4\pi} \frac{a}{(a^2 + z^2)^{3/2}} \hat{\mathbf{z}}$$

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