MUTUAL INDUCTANCE BETWEEN TWO LOOPS

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.21.

Here's a simple example of how the equality of mutual inductance between two circuits can make problem solving a bit easier. Suppose we have a small, square loop of side length a situated midway between two very long wires a distance $3a$ apart. The two long wires are connected at both ends to form a circuit, but we'll assume that these ends are far enough away that they don't influence what happens in the small square.

If the small square has a current that is increasing at a constant rate of $\dot{I}=k>0$, what emf is induced in the large loop?

Trying to work out the field generated by the small loop and calculate from that the change in flux in the large loop would be very difficult, so we can invert the problem by supposing that the current flows in the large loop and from that obtain the mutual inductance M.

The magnetic field produced by each of the long wires is

$$\mathbf{B}(r,t)=\hat{\phi}\frac{\mu_0 I(t)}{2\pi r} \tag{1}$$

where the ϕ direction is obtained by using the right-hand rule and the direction of current, and r is the distance from the wire. Since the little square is midway between the two wires, each wire contributes the same flux, and in the same direction (since the current in each of the two wires is opposite to that in the other wire), so since the little square spans a distance between a and $2a$ from each wire:

$$\Phi(t)=2\frac{\mu_0 I(t)}{2\pi}\int_a^{2a}\frac{a\,dr}{r}=\frac{\mu_0 a I(t)}{\pi}\ln 2 \tag{2}$$

The mutual inductance is then

$$M=\frac{\Phi}{I}=\frac{\mu_0 a}{\pi}\ln 2 \tag{3}$$

and the induced emf is

$$\mathscr{E} = -\frac{d\Phi}{dt} = -M\frac{dI}{dt} = -\frac{\mu_0 ak}{\pi}\ln 2 \tag{4}$$

This is also the emf induced in the large loop by a changing current in the small square. If the current in the small square is flowing clockwise, then the induced current in the large loop opposes the increase of current in the small square, so it will be counterclockwise.

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