

SELF-INDUCTANCE OF A LONG RECTANGLE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.23.

We'll return to the earlier problem in which a small loop was enclosed between a larger circuit consisting essentially of two very long wires a distance d apart, connected at the far ends. This time, we'll remove the small loop and try to find the self-inductance of the large loop. The magnetic field produced by each of the long wires is

$$\mathbf{B} = \hat{\phi} \frac{\mu_0 I}{2\pi r} \quad (1)$$

where the ϕ direction is obtained by using the right-hand rule and the direction of current, and r is the distance from the wire. Since the flux from each wire is $\Phi_w = \int \mathbf{B} \cdot d\mathbf{a} = LI$ it would seem that all we need to do is a simple integral to get the inductance. The problem, however, arises in the limits of the integral. The total flux is twice that due to each wire, so for a unit length, we get

$$\Phi = 2 \frac{\mu_0 I}{2\pi} \int_0^d \frac{dr}{r} \quad (2)$$

The integral gives a logarithm, which is infinite at the lower limit of 0. In reality, of course, no wire is infinitely thin, so if we give the wire a radius of ϵ , then we get

$$\Phi = 2 \frac{\mu_0 I}{2\pi} \int_{\epsilon}^{d-\epsilon} \frac{dr}{r} = \frac{\mu_0 I}{\pi} \ln \frac{d-\epsilon}{\epsilon} \quad (3)$$

so the inductance per unit length is

$$L = \frac{\mu_0}{\pi} \ln \frac{d-\epsilon}{\epsilon} \quad (4)$$