SELF-INDUCTANCE OF A LONG RECTANGLE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.23.

We'll return to the earlier problem in which a small loop was enclosed between a larger circuit consisting essentially of two very long wires a distance d apart, connected at the far ends. This time, we'll remove the small loop and try to find the self-inductance of the large loop. The magnetic field produced by each of the long wires is

$$\mathbf{B}=\hat{\phi}\frac{\mu_0 I}{2\pi r} \tag{1}$$

where the ϕ direction is obtained by using the right-hand rule and the direction of current, and r is the distance from the wire. Since the flux from each wire is $\Phi_w=\int\mathbf{B}\cdot d\mathbf{a}=LI$ it would seem that all we need to do is a simple integral to get the inductance. The problem, however, arises in the limits of the integral. The total flux is twice that due to each wire, so for a unit length, we get

$$\Phi=2\frac{\mu_0 I}{2\pi}\int_0^d\frac{dr}{r} \tag{2}$$

The integral gives a logarithm, which is infinite at the lower limit of 0. In reality, of course, no wire is infinitely thin, so if we give the wire a radius of ϵ, then we get

$$\Phi=2\frac{\mu_0 I}{2\pi}\int_{\epsilon}^{d-\epsilon}\frac{dr}{r}=\frac{\mu_0 I}{\pi}\ln\frac{d-\epsilon}{\epsilon} \tag{3}$$

so the inductance per unit length is

$$L=\frac{\mu_0}{\pi}\ln\frac{d-\epsilon}{\epsilon} \tag{4}$$