INDUCTION BETWEEN A WIRE AND A TORUS

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.24.

An interesting inductance problem consists of an infinite straight wire carrying an alternating current $I(t) = I_0 \cos \omega t$. This wire lies along the axis of a toroidal solenoid of rectangular cross-section, with inner radius a, outer radius b and height h, and with a total of N turns. The solenoid is connected in series with a resistor R. What emf is induced in the solenoid, and thus what current flows through the resistor?

The magnetic field due to this current is (using the quasistatic approximation, in which the current changes everywhere along the wire at exactly the same rate):

$$\mathbf{B} = \hat{\phi} \frac{\mu_0 I}{2\pi r} \tag{0.1}$$

The flux per turn in the solenoid is then

$$\Phi_1 = \frac{h\mu_0 I}{2\pi} \int_a^b \frac{dr}{r} = \frac{h\mu_0 I}{2\pi} \ln \frac{b}{a} \tag{0.2}$$

The total flux through the torus is then

$$\Phi = \frac{h\mu_0 IN}{2\pi} \ln \frac{b}{a} \tag{0.3}$$

The emf induced in the solenoid is

$$\mathscr{E} = -\frac{d\Phi}{dt} = \frac{h\mu_0 \dot{I} N}{2\pi} \ln \frac{b}{a} = \frac{h\mu_0 N}{2\pi} I_0 \omega \sin \omega t \ln \frac{b}{a} \tag{0.4}$$

The current through the resistor is then

$$I_r = \frac{\mathscr{E}}{R} = \frac{h\mu_0 N}{2\pi R} I_0 \omega \sin \omega t \ln \frac{b}{a} \tag{0.5}$$

Putting in some actual numbers, we set $a = 1$ cm, $b = 2$ cm, $h = .01$ m, $N = 1000$, $I_0 = 0.5$ A, $\omega = 60$ Hz $= 2\pi \times 60\ \mathrm{s}^{-1}$ and $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N} \cdot \mathrm{A}^{-2}$ we get

$$\mathscr{E}=2.613\times10^{-4}\sin\omega t\text{ Volts} \tag{0.6}$$

If the resistor is $R=500\ \Omega$ the current through it is

$$I_{r}=\frac{2.613\times10^{-4}\sin\omega t}{500}=5.23\times10^{-7}\sin\omega t\text{ Amp} \tag{0.7}$$

In other words, the voltage and current involved are very small.

The changing current within the solenoid generates a *back emf* because of the solenoid's self-inductance. In Griffiths's example 7.11, the inductance of a toroidal solenoid is given as

$$L=\frac{h\mu_{0}N^{2}}{2\pi}\ln\frac{b}{a}=1.386\times10^{-3}\text{ henries} \tag{0.8}$$

The flux through the solenoid due to the induced current is

$$\Phi_{b}=LI_{r}=7.25\times10^{-10}\sin\omega t \tag{0.9}$$

so the back emf is

$$\mathscr{E}_{b}=-\frac{d\Phi_{b}}{dt}=-2.73\times10^{-7}\cos\omega t\text{ V} \tag{0.10}$$

The ratio of back emf to forward emf is then

$$\frac{\mathscr{E}_{b}}{\mathscr{E}}=1.046\times10^{-3} \tag{0.11}$$

Thus the back emf is only a small fraction of the original induced emf.

(For those keeping track of such things, this is post #666.)

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