ENERGY IN A TOROIDAL SOLENOID

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.27.

Here's another simple example of calculating the energy in the magnetic field. This time we'll look at a toroidal solenoid with N turns carrying a current I. The field is given in Griffiths's example 5.10:

$$\mathbf{B}=\frac{\mu_0 NI}{2\pi r}\hat{\phi} \tag{1}$$

This formula applies to a torus with an arbitrary cross section, but we'll look at a rectangular cross section here. The inner radius is a, outer radius b and height h. The energy is

$$\begin{aligned} W_B &= \frac{1}{2\mu_0}\int B^2 d^3\mathbf{r} & (2)\\ &= \frac{1}{2\mu_0}\left(\frac{\mu_0 NI}{2\pi}\right)^2\int_a^b \frac{2\pi rh}{r^2}dr & (3)\\ &= \frac{\mu_0 N^2 I^2 h}{4\pi}\ln\frac{b}{a} & (4)\end{aligned}$$

Since $W_B=\frac{1}{2}LI^2$ we can use this formula to find the inductance of the torus.

$$L=\frac{\mu_0 N^2 h}{2\pi}\ln\frac{b}{a} \tag{5}$$

which agrees with Griffiths's equation 7.27 in example 7.11.