

ENERGY IN A TOROIDAL SOLENOID

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.27.

Here's another simple example of calculating the energy in the magnetic field. This time we'll look at a toroidal solenoid with N turns carrying a current I . The field is given in Griffiths's example 5.10:

$$(1) \quad \mathbf{B} = \frac{\mu_0 N I}{2\pi r} \hat{\phi}$$

This formula applies to a torus with an arbitrary cross section, but we'll look at a rectangular cross section here. The inner radius is a , outer radius b and height h . The energy is

$$(2) \quad W_B = \frac{1}{2\mu_0} \int B^2 d^3\mathbf{r}$$
$$(3) \quad = \frac{1}{2\mu_0} \left(\frac{\mu_0 N I}{2\pi} \right)^2 \int_a^b \frac{2\pi r h}{r^2} dr$$
$$(4) \quad = \frac{\mu_0 N^2 I^2 h}{4\pi} \ln \frac{b}{a}$$

Since $W_B = \frac{1}{2} L I^2$ we can use this formula to find the inductance of the torus.

$$(5) \quad L = \frac{\mu_0 N^2 h}{2\pi} \ln \frac{b}{a}$$

which agrees with Griffiths's equation 7.27 in example 7.11.