LR CIRCUIT

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.29.

Suppose we have a circuit consisting of a battery delivering a voltage $\mathcal{E}_0$, an inductor L and a resistor R, all in series. If the circuit has been connected long enough to stabilize, suppose we then throw a switch that bypasses the battery and isolates L and R in their own loop. The inductor will then produce some back emf that opposes the drop in current, in the amount $\mathcal{E} = -L\dot{I}$. Since this is now the only source of emf in the circuit, it must equal the voltage drop across the resistor, so

$$-L\dot{I} = IR \tag{0.1}$$

$$\dot{I} = -\frac{R}{L}I \tag{0.2}$$

$$I(t) = I_0 e^{-Rt/L} = \frac{\mathcal{E}_0}{R}e^{-Rt/L} \tag{0.3}$$

where I_0 is the initial current, which is $I_0 = \mathcal{E}_0/R$.

The total energy delivered to the resistor by the inductor is

$$W = R\int_0^\infty I^2(t)\,dt = I_0^2 R\int_0^\infty e^{-2Rt/L}dt = \frac{1}{2}LI_0^2 \tag{0.4}$$

From the energy formula for an inductor, this is equal to the initial energy in the inductor's magnetic field.

$$W = \frac{1}{2}L\left(\frac{\mathcal{E}_0}{R}\right)^2 \tag{0.5}$$