DISPLACEMENT CURRENT IS VERY SMALL

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.33.

We'll consider again the problem of induced magnetic and electric fields in a coaxial cable with alternating current. A thin wire carries current $I(t) = I_0 \cos\omega t$ and the current returns down the coaxial cylinder of radius a. We saw earlier that the induced electric field is

$$\mathbf{E}(s,t) = \begin{cases} \hat{\mathbf{z}}\frac{\omega\mu_0 I_0}{2\pi}\sin\omega t\ln\frac{a}{s} & 0 < s < a \\ 0 & s > a \end{cases} \tag{1}$$

The displacement current density inside the cylinder is then

$$\mathbf{J}_d = \varepsilon_0 \frac{\partial\mathbf{E}}{\partial t} = \hat{\mathbf{z}}\frac{\omega^2\mu_0 I_0\varepsilon_0}{2\pi}\cos\omega t\ln\frac{a}{s} \tag{2}$$

The total displacement current is then

$$\begin{aligned} I_d &= \frac{\omega^2\mu_0 I_0\varepsilon_0}{2\pi}\cos\omega t\int_0^a 2\pi s\ln\frac{a}{s}ds & (3)\\ &= \omega^2\mu_0 I_0\varepsilon_0\cos\omega t\left[\frac{s^2}{2}\ln\frac{a}{s}+\frac{s^2}{4}\right]_0^a & (4)\\ &= \frac{\omega^2\mu_0 I_0\varepsilon_0 a^2}{4}\cos\omega t & (5)\end{aligned}$$

The ratio of the two currents is

$$\frac{I_d}{I} = \frac{\omega^2\mu_0\varepsilon_0 a^2}{4} \tag{6}$$

We can convert this to a numerical value if we note that $\mu_0\varepsilon_0 = 1/c^2 = 1/\left(3\times10^8\right)^2$ so

$$\frac{I_d}{I} = 2.78\times10^{-18}\omega^2 a^2 \tag{7}$$

If $a = 1$ mm, then to get I_d up to 1% of I, we would need a frequency of

$$\omega^2 = \frac{(0.01)}{(2.78\times10^{-18})\,(10^{-3})^2} \tag{8}$$

$$\omega = 6\times10^{10}\ \mathrm{s}^{-1} \tag{9}$$

or in cycles per second

$$\nu = 9.55\times10^{9}\ \mathrm{Hz} \tag{10}$$

Ordinary household current is in the range 50 - 60 Hz, but 10^{10} Hz is in the microwave region of the EM spectrum.