

## DISPLACEMENT CURRENT IS VERY SMALL

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.33.

We'll consider again the problem of induced magnetic and electric fields in a coaxial cable with alternating current. A thin wire carries current  $I(t) = I_0 \cos \omega t$  and the current returns down the coaxial cylinder of radius  $a$ . We saw earlier that the induced electric field is

$$\mathbf{E}(s, t) = \begin{cases} \hat{\mathbf{z}} \frac{\omega \mu_0 I_0}{2\pi} \sin \omega t \ln \frac{a}{s} & 0 < s < a \\ 0 & s > a \end{cases} \quad (1)$$

The displacement current density inside the cylinder is then

$$\mathbf{J}_d = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \hat{\mathbf{z}} \frac{\omega^2 \mu_0 I_0 \epsilon_0}{2\pi} \cos \omega t \ln \frac{a}{s} \quad (2)$$

The total displacement current is then

$$I_d = \frac{\omega^2 \mu_0 I_0 \epsilon_0}{2\pi} \cos \omega t \int_0^a 2\pi s \ln \frac{a}{s} ds \quad (3)$$

$$= \omega^2 \mu_0 I_0 \epsilon_0 \cos \omega t \left[ \frac{s^2}{2} \ln \frac{a}{s} + \frac{s^2}{4} \right]_0^a \quad (4)$$

$$= \frac{\omega^2 \mu_0 I_0 \epsilon_0 a^2}{4} \cos \omega t \quad (5)$$

The ratio of the two currents is

$$\frac{I_d}{I} = \frac{\omega^2 \mu_0 \epsilon_0 a^2}{4} \quad (6)$$

We can convert this to a numerical value if we note that  $\mu_0 \epsilon_0 = 1/c^2 = 1/(3 \times 10^8)^2$  so

$$\frac{I_d}{I} = 2.78 \times 10^{-18} \omega^2 a^2 \quad (7)$$

If  $a = 1$  mm, then to get  $I_d$  up to 1% of  $I$ , we would need a frequency of

$$\omega^2 = \frac{(0.01)}{(2.78 \times 10^{-18})(10^{-3})^2} \quad (8)$$

$$\omega = 6 \times 10^{10} \text{ s}^{-1} \quad (9)$$

or in cycles per second

$$\nu = 9.55 \times 10^9 \text{ Hz} \quad (10)$$

Ordinary household current is in the range 50 - 60 Hz, but  $10^{10}$  Hz is in the microwave region of the EM spectrum.