

DISPLACEMENT CURRENT IS VERY SMALL

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.33.

We'll consider again the problem of induced magnetic and electric fields in a coaxial cable with alternating current. A thin wire carries current $I(t) = I_0 \cos \omega t$ and the current returns down the coaxial cylinder of radius a . We saw earlier that the induced electric field is

$$(1) \quad \mathbf{E}(s,t) = \begin{cases} \hat{\mathbf{z}} \frac{\omega \mu_0 I_0}{2\pi} \sin \omega t \ln \frac{a}{s} & 0 < s < a \\ 0 & s > a \end{cases}$$

The displacement current density inside the cylinder is then

$$(2) \quad \mathbf{J}_d = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \hat{\mathbf{z}} \frac{\omega^2 \mu_0 I_0 \epsilon_0}{2\pi} \cos \omega t \ln \frac{a}{s}$$

The total displacement current is then

$$(3) \quad I_d = \frac{\omega^2 \mu_0 I_0 \epsilon_0}{2\pi} \cos \omega t \int_0^a 2\pi s \ln \frac{a}{s} ds$$

$$(4) \quad = \omega^2 \mu_0 I_0 \epsilon_0 \cos \omega t \left[\frac{s^2}{2} \ln \frac{a}{s} + \frac{s^2}{4} \right]_0^a$$

$$(5) \quad = \frac{\omega^2 \mu_0 I_0 \epsilon_0 a^2}{4} \cos \omega t$$

The ratio of the two currents is

$$(6) \quad \frac{I_d}{I} = \frac{\omega^2 \mu_0 \epsilon_0 a^2}{4}$$

We can convert this to a numerical value if we note that $\mu_0 \epsilon_0 = 1/c^2 = 1/(3 \times 10^8)^2$ so

$$(7) \quad \frac{I_d}{I} = 2.78 \times 10^{-18} \omega^2 a^2$$

If $a = 1$ mm, then to get I_d up to 1% of I , we would need a frequency of

$$(8) \quad \omega^2 = \frac{(0.01)}{(2.78 \times 10^{-18})(10^{-3})^2}$$

$$(9) \quad \omega = 6 \times 10^{10} \text{ s}^{-1}$$

or in cycles per second

$$(10) \quad \nu = 9.55 \times 10^9 \text{ Hz}$$

Ordinary household current is in the range 50 - 60 Hz, but 10^{10} Hz is in the microwave region of the EM spectrum.