MAGNETIC MONOPOLE FORCE LAW

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.35.

If magnetic monopoles did exist, we could consider the behaviour of magnetic charges q_m. We could postulate a Coulomb's law for them:

$$\mathbf{F}=\frac{\mu_0}{4\pi}\frac{q_{m_1}q_{m_2}(\mathbf{r}_2-\mathbf{r}_1)}{|\mathbf{r}_2-\mathbf{r}_1|^3} \tag{1}$$

Griffiths sets the problem of 'working out' the force law for a magnetic charge passing through electric and magnetic fields. This strikes me as rather difficult, since the Lorentz force law for electric charges is essentially a postulate of the theory (confirmed by experiment); it was never derived. The simplest equivalent magnetic force law would seem to be something like

$$\mathbf{F}=q_m\mathbf{B}+q_m\mathbf{v}\times\mathbf{E} \tag{2}$$

We do need to check the units, however. First, what are the units of a magnetic charge q_m? The units of μ_0 are m kg s^{-2} A^{-2} so the Coulomb force law gives us

$$\text{kg m s}^{-2}=\text{m kg s}^{-2}\ \text{A}^{-2}\,[q_m]^2\,\text{m}^{-2} \tag{3}$$

so the units of q_m must be A m = Coulomb m s^{-1}, that is, charge times velocity. How does this fit with our proposed Lorentz-like force law? The units of $\mathbf{B}$ are kg m s^{-2} A^{-1} m^{-1}, so $q_m\mathbf{B}$ has the units of force, so that term seems OK.

For the second term, we know that electric charge times electric field give force, so that means that $q_m\mathbf{v}\times\mathbf{E}$ has the units of force times velocity squared, so we need to divide by some constant velocity squared. Since we know that $\mu_0\epsilon_0=1/c^2$ this seems a logical choice, so we can propose

$$\mathbf{F}=q_m\mathbf{B}+\mu_0\epsilon_0 q_m\mathbf{v}\times\mathbf{E}=q_m\mathbf{B}+\frac{q_m}{c^2}\mathbf{v}\times\mathbf{E} \tag{4}$$

In fact, the commonly accepted form is

$$\mathbf{F}=q_m\mathbf{B}-\frac{q_m}{c^2}\mathbf{v}\times\mathbf{E} \tag{5}$$

I could not find a 'derivation' of this result; rather it seems that this form is adopted to make this force law invariant under a duality transformation.

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