

MAGNETIC MONOPOLE FORCE LAW

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.35.

If magnetic monopoles did exist, we could consider the behaviour of magnetic charges q_m . We could postulate a Coulomb's law for them:

$$(0.1) \quad \mathbf{F} = \frac{\mu_0 q_{m1} q_{m2} (\mathbf{r}_2 - \mathbf{r}_1)}{4\pi |\mathbf{r}_2 - \mathbf{r}_1|^3}$$

Griffiths sets the problem of 'working out' the force law for a magnetic charge passing through electric and magnetic fields. This strikes me as rather difficult, since the Lorentz force law for electric charges is essentially a postulate of the theory (confirmed by experiment); it was never derived. The simplest equivalent magnetic force law would seem to be something like

$$(0.2) \quad \mathbf{F} = q_m \mathbf{B} + q_m \mathbf{v} \times \mathbf{E}$$

We do need to check the units, however. First, what are the units of a magnetic charge q_m ? The units of μ_0 are $\text{m kg s}^{-2} \text{A}^{-2}$ so the Coulomb force law gives us

$$(0.3) \quad \text{kg m s}^{-2} = \text{m kg s}^{-2} \text{A}^{-2} [q_m]^2 \text{m}^{-2}$$

so the units of q_m must be $\text{A m} = \text{Coulomb m s}^{-1}$, that is, charge times velocity. How does this fit with our proposed Lorentz-like force law? The units of \mathbf{B} are $\text{kg m s}^{-2} \text{A}^{-1} \text{m}^{-1}$, so $q_m \mathbf{B}$ has the units of force, so that term seems OK.

For the second term, we know that electric charge times electric field give force, so that means that $q_m \mathbf{v} \times \mathbf{E}$ has the units of force times velocity squared, so we need to divide by some constant velocity squared. Since we know that $\mu_0 \epsilon_0 = 1/c^2$ this seems a logical choice, so we can propose

$$(0.4) \quad \mathbf{F} = q_m \mathbf{B} + \mu_0 \epsilon_0 q_m \mathbf{v} \times \mathbf{E} = q_m \mathbf{B} + \frac{q_m}{c^2} \mathbf{v} \times \mathbf{E}$$

In fact, the commonly accepted form is

$$(0.5) \quad \mathbf{F} = q_m \mathbf{B} - \frac{q_m}{c^2} \mathbf{v} \times \mathbf{E}$$

I could not find a 'derivation' of this result; rather it seems that this form is adopted to make this force law invariant under a duality transformation.

PINGBACKS

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