DETECTING MAGNETIC MONOPOLES

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.36.

Blas Cabrera (1982), Physical Review Letters, **48**, 1378.

Although magnetic monopoles have never been found (at least, reproducibly), one experiment that tried to find them was that of Cabrera in 1982. The experiment was a fairly simple setup, consisting of a superconducting (and therefore zero-resistance) wire loop with a magnet aligned so that monopoles, should they exist, could pass through the loop. Assuming that a single magnetic 'charge' q_m emits a magnetic field that obeys a Coulomb-like law, that is

$$\mathbf{B}=\frac{\mu_0}{4\pi}\frac{q_m}{r^2}\hat{\mathbf{r}} \tag{1}$$

we can work out the magnetic flux through the loop as a single monopole falls through it.

If the speed of the monopole is v and it falls along the axis of the loop then, assuming the loop has radius b and we take as the area of integration the flat circle within the loop, we need to work out $\int\mathbf{B}\cdot d\mathbf{a}$ to get the flux. Suppose the monopole is a distance d from the centre of the loop. Then the distance from the monopole to a point on the disk with radius s is $\sqrt{d^2+s^2}$ so the field strength at that point is

$$B=\frac{\mu_0}{4\pi}\frac{q_m}{d^2+s^2} \tag{2}$$

The term $\mathbf{B}\cdot d\mathbf{a}$ isolates the component of $\mathbf{B}$ that is perpendicular to the disk, which is $B\cos\theta$ where θ is the angle between the axis and a line from the monopole to a point on the disk at radius s. We get

$$\cos\theta=\frac{d}{\sqrt{d^2+s^2}} \tag{3}$$

so the flux from the monopole is

$$\Phi = \int \mathbf{B}\cdot d\mathbf{a} \tag{4}$$

$$= \frac{\mu_0 q_m}{4\pi}\int_0^b \frac{2\pi s d}{\left(d^2+s^2\right)^{3/2}} ds \tag{5}$$

$$= -\frac{\mu_0 q_m}{2}\left.\frac{d}{\sqrt{d^2+s^2}}\right|_0^b \tag{6}$$

$$= \frac{\mu_0 q_m}{2}\left[1-\frac{d}{\sqrt{d^2+b^2}}\right] \tag{7}$$

If we take $t=0$ to be the time when the monopole crosses the plane of the disk and take the surface normal at the disk to point in the direction of the monopole's velocity, then $d=-vt$ for $t<0$ and $d=+vt$ for $t>0$, and $\mathbf{B}\cdot d\mathbf{a}>0$ for $t<0$ and $\mathbf{B}\cdot d\mathbf{a}<0$ for $t>0$. That is

$$\Phi(t) = \begin{cases} \frac{\mu_0 q_m}{2}\left[1+\frac{vt}{\sqrt{(vt)^2+b^2}}\right] & t<0 \\ -\frac{\mu_0 q_m}{2}\left[1-\frac{vt}{\sqrt{(vt)^2+b^2}}\right] & t>0 \end{cases} \tag{8}$$

To go further, we need to modify Maxwell's equations to include magnetic charge. The relevant one is Faraday's law which needs an extra term:

$$\nabla\times\mathbf{E} = -\mu_0\mathbf{J}_m - \frac{\partial\mathbf{B}}{\partial t} \tag{9}$$

where $\mathbf{J}_m$ is the magnetic current density. This is the analog to the $\nabla\times\mathbf{B}$ equation which involves electric current density. Applying Stokes's theorem, we can integrate the LHS around the loop and the RHS over the disk enclosed:

$$\oint \mathbf{E}\cdot d\ell = -\mu_0\int \mathbf{J}_m\cdot d\mathbf{a} - \int \frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{a} \tag{10}$$

$$\mathscr{E} = -\mu_0 I_{m-enc} - \frac{d\Phi}{dt} \tag{11}$$

where $\mathscr{E}$ is the induced back emf and I_{m-enc} is the magnetic current flowing through the loop. This emf can be written in terms of the self-inductance L of the loop:

$$\mathscr{E} = -L\frac{dI(t)}{dt} \tag{12}$$

What are we to make of I_{m-enc} considering we have only a single monopole to make up the current? We can write it as a delta function:

$$I_{m-enc} = q_m \delta(t) \tag{13}$$

That is, there is a current consisting of a single charge across the disk only at time $t = 0$. Since the delta function is the derivative of the step-function H, we can integrate Faraday's law to get

$$LI(t) = \mu_0 q_m H(t) + \Phi(t) \tag{14}$$

So we get

$$I(t) = \begin{cases} \frac{\mu_0 q_m}{2L}\left[1 + \frac{vt}{\sqrt{(vt)^2+b^2}}\right] & t < 0 \\ \frac{\mu_0 q_m}{L} - \frac{\mu_0 q_m}{2L}\left[1 - \frac{vt}{\sqrt{(vt)^2+b^2}}\right] & t > 0 \end{cases} \tag{15}$$

The $t > 0$ term comes out to be the same as the $t < 0$ term, so we get in general

$$I(t) = \frac{\mu_0 q_m}{2L}\left[1 + \frac{vt}{\sqrt{(vt)^2 + b^2}}\right] \tag{16}$$

For $t \to -\infty$, $I \to 0$, while for $t \to \infty$, $I \to \mu_0 q_m / L$. This means that, when the monopole is infinitely far away and approaching, there is no induced current. The current increases as the monopole gets closer, but since the superconducting loop has no resistance, the current that is built up due to the back emf never dissipates and tends to a constant non-zero value as the monopole recedes into the distance.